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A hybrid time series forecasting method based on neutrosophic logic with applications in financial issues

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ABSTRACT

Rising market demands, economic pressures, and technological advancements have spurred researchers to seek ways to enhance business environments and scientific productivity. Predictive science, crucial in this context, has gained prominence due to the rapid progress in information technology and forecasting algorithms. Time series forecasting, widely used in fields like engineering, economics, tourism, and energy, has inherent limitations with classical statistical methods, leading researchers to explore artificial intelligence and fuzzy logic for more accurate predictions. However, despite extensive efforts to improve accuracy, challenges persist. The research introduces a model aimed at surpassing existing methods in time series forecasting accuracy. This approach combines meta-heuristic optimization algorithms and neutrosophic logic to enhance precision in uncertain and complex environments, promising improved forecasting outcomes. The study shows that the performance of the neutrosophic time series modeling approach is highly dependent on the optimal selection of the universe of discourse and its corresponding intervals. This study selects the quantum optimization algorithm (QOA), genetic algorithm (GA), and particle swarm optimization (PSO) to address this weakness. These optimization algorithms improve the performance of the NTS modeling approach by selecting the global universe of discourse and corresponding intervals from the list of locally optimal solutions. The proposed hybrid model (i.e., NTS-QOA model) is verified and validated with datasets of university enrollment of Alabama (USA), Taiwan futures exchange (TAIFEX) index, and Taiwan Stock Exchange Corporation (TSEC) weighted index. Various experimental results signified the efficiency of the proposed model over existing benchmark models in terms of average forecasting error rate (AFER). This value using the proposed NTS QOA, NTS GA, and NTS PSO method on the university dataset is 0.166, 0.167, 0.164, on the TAIFEX dataset, is 0.081, 0.081, and 0.081, and on the TSEC dataset is 0.09, 0.09, and 0.09, respectively.

1. Introduction

Researchers have been driven by the urgent demands of the market, economy, and production industries, as well as the need to keep up with advancing technology to seek out ways to enhance the business environment and scientific production (Shemshad and Karim, 2023). Among these efforts, forecasting science is particularly essential (Taghipourian et al., 2021). Forecasting involves predicting future events based on scientific and logical principles and rules, making it a crucial tool in many contexts. We require information and data and practical and rational methods for data analysis to achieve accurate predictions (Shoaee and Gholi Keshmarzi, 2023).

Forecasting is essential for effective planning, regardless of the circumstances or time horizon. With the rapid advancement of information technology and forecasting algorithms, relying solely on existing methods may no longer suffice (Kuranga et al., 2023). In today's world,

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informed decision-making requires accurate forecasting, and time series forecasting is a widely studied method with applications across various fields such as engineering (Abbasimehr and Khodizadeh Nahari, 2020), economics (Udoka, 2020), (Fakhrehosseini and Kaviani, 2023), tourism (Dong et al., 2023), and energy (Guo et al., 2023). Precise analysis and forecasting of data provide valuable insights to researchers. While classical statistical methods are commonly used for predicting raw statistical data, they may have unavoidable inaccuracies (Etemadi and Khashei, 2023), (Casal-Guisande et al., 2023). Hence, researchers have turned to prediction methods based on artificial intelligence and fuzzy logic for more precise outputs.

Time series forecasting involves selecting the optimal model parameters and settings to achieve accurate predictions (Abdolhosseini, 2023). Time series data often contain inherent uncertainty and complexity, which classical statistical methods may struggle to address effectively (ADEBISI and Smarandache, 2022). Metaheuristic algorithms are well-suited to handle such complex and uncertain environments (Espinosa et al., 2023). Metaheuristic algorithms, such as the Quantum Optimization Algorithm (QOA), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO), are designed to search for optimal solutions in complex and high-dimensional spaces. Traditional statistical methods and optimization techniques may find local optima and struggle to explore the entire solution space (Eyo et al., 2021). Metaheuristic algorithms are designed to escape local optima and search for global optima, which is crucial for identifying the best combination of parameters for the NTS model. This global optimization aspect is particularly important in improving forecasting accuracy. Metaheuristic algorithms automate the parameter selection process, reducing the need for manual tuning and experimentation. This automation improves the efficiency of the forecasting process, making it more practical for real-world applications.

The study employs GA, PSO, and QOA for specific reasons, and each of these algorithms has advantages over other optimization techniques in the context of the research objectives: GA is a widely recognized and powerful optimization algorithm that is particularly effective in searching for global optima in complex, high-dimensional search spaces. It is known for its ability to explore a wide range of potential solutions, making it suitable for problems with numerous parameters. GA is also highly adaptable and can be customized to various problem domains (Sohail, 2023). GA global optimization capabilities are valuable in ensuring the chosen parameters are well-suited to the forecasting task. PSO is another popular optimization technique, particularly for continuous parameter optimization. It is known for its simplicity and ease of implementation. It is often computationally efficient and can quickly converge to optimal solutions (Daneshvar et al., 2023). The advantage of PSO lies in its efficiency and its ability to handle continuous search spaces (Etebari et al., 2021). QOA is a relatively newer optimization algorithm inspired by quantum computing principles. QOA has shown promise in handling optimization problems by leveraging quantum principles, which can lead to potentially superior solutions for certain problems. QOA introduces a cutting-edge dimension to the research, showcasing the potential of quantum-inspired optimization techniques in improving the forecasting model's accuracy.

While other optimization algorithms could be considered, the use of GA, PSO, and QOA aligns well with the research objectives, offering a combination of global search capabilities, efficiency, and potential advancements in optimization for the specific problem of parameter selection in Neutrosophic Time Series (NTS) modeling.

Despite significant efforts to improve existing models' prediction abilities, enhancing accuracy remains complex and challenging (Hajirahimi and Khashei, 2023). Over the past few decades, fuzzy sets have been utilized to predict time series (Abbaspour Ghadim Bonab, 2022). However, their limitations lie in the fact that they only express a phenomenon's membership level, neglecting numerous uncertain concepts (Qiu et al., 2023). Generalized fuzzy systems such as intuitive, interval, and Pythagorean fuzzy were developed to overcome this issue (Martin and Edalatpanah, 2023). While these tools have reasonable efficiency compared to fuzzy systems, they cannot measure the concept of indeterminism (Saberhoseini et al., 2022). Our world is full of different phenomena - some are true, some are false, and others cannot be easily classified. Therefore, Smarandache introduced neutrosophic logic, currently considered the most comprehensive measurement tool in uncertain and complex environments.

Neutrosophic sets extend the representation of uncertainty beyond what traditional fuzzy sets offer. While traditional fuzzy sets express membership and non-membership degrees, neutrosophic sets add a third component: indeterminacy. This three-degree membership concept allows for a more comprehensive and nuanced representation of uncertainty, which is particularly useful when dealing with complex and ambiguous data in time series forecasting (Veeramani et al., 2022). In many real-world situations, data is not just a matter of being true or false; there is often an indeterminate or uncertain aspect. Neutrosophic sets excel at capturing and managing this indeterminacy, a common feature in time series data.

The neutrosophic theory is particularly effective in handling complex and uncertain scenarios. It can represent situations where a phenomenon is partly true, partly false, and somewhat indeterminate. These situations are often the case in time series data, where some values may be imprecise, uncertain, or fluctuating. Other generalizations of fuzzy sets, such as interval-valued or intuitionistic fuzzy sets, do not provide as comprehensive a framework for dealing with this aspect of uncertainty (Ihsan et al., 2023).

The challenge in earlier studies, which this research aims to address, is the limited capability of existing time series forecasting models, particularly those based on traditional statistical methods or fuzzy logic, to effectively capture and model uncertainty and complexity in time series data.

Earlier studies often relied on fuzzy logic-based time series forecasting models (Hieu et al., 2023), primarily dealing with the membership degree of elements in a set. For instance, traditional Fuzzy Time Series (FTS) models could handle concepts like "very likely" or "unlikely." However, they struggled to represent and handle indeterminate data points or had multiple degrees of membership. As a result, these models couldn't effectively capture complex patterns in data where elements may be partly true, partly false, or entirely uncertain (Pramanik et al., 2023).

Prior research sometimes focused on time series classification (Zarghami et al., 2023), where the goal was to assign a category or label to a time series based on historical data. While these approaches were effective for classification tasks, they didn't necessarily address the challenge of accurate time series prediction or forecasting, where the goal is to predict future values or trends. Earlier studies often lacked real-world applications or demonstrations of the proposed models' effectiveness in practical scenarios, limiting the ability to assess the models' impact on real decision-making and planning.

The examples provided highlight the challenge that earlier studies faced in effectively modeling and addressing uncertainty, indeterminacy, and complexity in time series data, as well as the limitation of their real-world applicability. This research seeks to overcome these limitations by introducing neutrosophic logic and employing advanced optimization algorithms, thus advancing the field of time series forecasting.

The current study offers several practical advantages over existing literature in time series forecasting. The study demonstrates that the proposed NTS-QOA model outperforms existing benchmark models regarding Average Forecasting Error Rate (AFER). It means that the model provides more accurate predictions, which can be highly beneficial in real-world applications where precise forecasting is crucial for decision-making. Integrating neutrosophic logic allows for a more comprehensive representation of uncertainty in time series data. Considering indeterminacy also goes beyond fuzzy logic's binary membership/non-membership representation. This characteristic is valuable when data is inherently uncertain and ambiguous.

The study applies the proposed approach to various datasets, including university enrollment data, financial indices, and stock exchange data. This versatility suggests that the model can be applied to a wide range of domains, making it a practical choice for different industries and research areas. Incorporating meta-heuristic optimization algorithms (QOA, GA, and PSO) enables the model to automatically select the most suitable universe of discourse and intervals for forecasting. It improves accuracy and streamlines forecasting, making it more practical for real-world applications. The enhanced accuracy of the forecasting model can provide decision-makers with more reliable insights. Accurate predictions are crucial for strategic planning, resource allocation, and risk management in business and other fields. The study's approach can serve as a valuable tool for informed decision-making.

The study's contributions are as follows.

A. The study acknowledges the limitations of previous time series forecasting models based on fuzzy logic and their inability to capture indeterminacy effectively.
B. The study introduces a novel approach that combines neutrosophic logic with time series forecasting. This integration allows for a more comprehensive representation of uncertainty in time series data, including true, indeterminate, and false components, which goes beyond the capabilities of traditional fuzzy logic.
C. The research incorporates three meta-heuristic optimization algorithms: the QOA, GA, and PSO. These algorithms enhance the accuracy of the NTS modeling approach by selecting the optimal universe of discourse and intervals for better predictions.
D. The proposed hybrid model, NTS-QOA, outperforms existing benchmark models in terms of AFER on various datasets, including university enrollment, financial indices, and stock exchange data. The study demonstrates that the NTS-QOA model offers higher precision in forecasting.

1.1. Related concepts

1.1.1. Time series

Time series refers to the collection of data and events that occur over a specific period, often presented as a sequence of regular observations, such as weekly oil or stock prices. The primary objectives of time series analysis include data description, modeling, forecasting (the most critical goal), and control. Typically, time series are represented as follows:

$X(t),\ T = 1, 2, 3, \ldots$

Where X(t) represents the random variable X at time t.

1.1.2. Fuzzy set theory

Uncertainty is a common aspect of events that occur around us. Classic approaches remove uncertainty from the real world and are not always applicable (Li et al., 2023). Therefore, Scientists have extensively utilized fuzzy sets to address uncertainty-related problems.

Definition 1. (Zadeh, 1965): Suppose X is given, then the fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\ |x \in X\}$, where $\mu\tilde{A}\ (x)$ is the membership function of the fuzzy set $\tilde{A}$. The membership function maps each member of X to a degree of membership that is a value between 0 and 1.

Despite the success of fuzzy sets in handling uncertainties arising from partial belonging or ambiguity within a set, they fall short in modeling all the uncertainty states prevalent in real-life problems, particularly those involving insufficient information. In light of this, Atanassov (1986) proposed a generalized version of fuzzy sets, known as intuitionistic fuzzy sets.

Definition 2. (Atanassov, 1986): An intuitionistic fuzzy set in U is a set like A in which two degrees are attributed to each member $u \in U$; one is the "degree of membership," and the other is the "degree of non-membership."

If the membership function is $\mu(A){:}X \rightarrow [0,1]$ and the non-membership function is $\nu\ (A){:}\ X \rightarrow [0,1]$, then the indeterminacy value of A is $1 - \mu(A) - \nu\ (A)$, and condition $0 \leq \mu(A) + \nu\ (A) \leq 1$ is always satisfied.

These types of sets not only have the degree of membership and non-membership but also consider the degree of indeterminacy and inconsistency. Accordingly, the function "Truth Membership" T(x), "Indeterminacy Membership" I(x), and the Falsehood Membership of F(x) are considered.

Definition 3. (Smarandache, 1998): A neutrosophic set is mapped as $A = \{<x, T_A(x), I_A(x), F_A(x)>|x \in X\}$, T_A: $X\rightarrow[0,1]$, I_A: $X\rightarrow[0,1]$, F_A: $X\rightarrow[0,1]$, and condition $0\leq T_A(x) + I_A(x) + F_A(x)\leq 3$ is always satisfied.

1.1.3. Fuzzy time series

Unlike traditional time series that work with real numbers, Fuzzy Time Series (FTS) have become famous for predicting dynamic and non-linear data sets in various fields, including stock markets (Chen and Chen, 2015). Over the years, several models for forecasting time series have been introduced in the scientific world, such as the famous Box-Jenkins model and ARIMA. However, these classical models face challenges in predicting linguistic variables and require vast data for accurate predictions. To overcome these challenges, Song & Chissom (Sun et al., 2015) proposed the FTS model, which consists of six steps: defining and dividing the reference set, defining fuzzy sets for observations, fuzzing observations, expressing fuzzy relations, making predictions, and defuzzifying observation results. These models have proven effective in solving forecasting problems in various fields, especially in accurately predicting stock prices.

1.1.4. Meta-heuristic algorithms

Experts utilize approximate algorithms to tackle intricate optimization problems. Such algorithms efficiently produce nearly optimal solutions within a short time frame and can be grouped into two types: Heuristic and algorithms. While heuristic algorithms may get trapped in local optima and cannot resolve diverse problems, meta-heuristic algorithms were designed to overcome these limitations (Sharifi et al., 2022). Essentially, meta-heuristic algorithms are probabilistic optimization algorithms that offer solutions not limited to local optima and can be implemented for a broad range of problems (Abdi et al., 2021).

1.1.5. Genetic algorithms (GA)

During the 1970s, John Holland, a scientist from the University of Michigan, proposed utilizing GA for engineering optimization. Other scientists, including Goldberg and Dejong, later expanded on this algorithm's application. The GA draws inspiration from the natural processes of living organisms and can be classified as a numerical method, direct search, and random search. The algorithm is based on iteration and borrows from principles observed in natural evolution to enhance solutions using the information within a population effectively.

1.1.6. Particle swarm optimization (PSO)

For the first time in 1995, Kennedy and Eberhart proposed their technique by modeling the movement of birds in the air, discovering the logical relationship between the change of direction and speed of birds, and using the knowledge of physics. These scientists realized the dependence of these movements on each other in their research and found that the movement of a bird is caused by the information it receives from the birds around it. Therefore, they completed the presented method and named it swarm movement. The cumulative movement algorithm of particles does not use operators such as intersection and mutation. As a result, it does not need to use strings of numbers and its coding stage. Therefore, it is much simpler than algorithms such as genetics. This algorithm divides the solution space into multi-piece paths

using a pseudo-probability function formed by the movement of individual particles in space. The motion of a group of particles consists of two main components: the deterministic component and the probabilistic component. Each particle is interested in moving toward the current best solution, x*, or the best solution obtained so far, g*.

1.1.7. Quantum optimization algorithm (QOA)

In quantum computing, the entanglement of quantum particles is essential to quantum mechanics. This unique property of quantum has no parallel in classical mechanics. Therefore, if the energy between two particles was high at any time in the past, those two particles can be considered intertwined (Melkikh, 2017). But none of these two particles are involved in the emission of other particles. Entanglement is commonly used in quantum communication, cryptography, and quantum computing (Melkikh, 2015). Schrödinger proposed the concept of "entanglement" in quantum mechanics (Melkikh, 2017) (Melkikh, 2015). Singh et al. (2018) proposed the QOA based on this quantum mechanics.

2. Literature review

Researchers have argued that traditional methods are less effective in predicting electricity production (EP) time series due to their nonstationary and nonlinearity. Therefore, they have proposed a novel combination prediction model based on wavelet transform (WT), long short-term memory (LSTM), and stacked autoencoder (SAE). Empirical results have shown that the combination model, specifically SAE-LSTM, outperforms benchmark models. Furthermore, the results suggest that WT-SAE-LSTM is superior to EMD, EEMD-SAE-LSTM, and SAE (Qiao et al., 2022).

A research paper presents a new hybrid forecasting model for short-term wind power forecasting. The model combines variational mode decomposition (VMD), two deep learning models - long short-term memory neural networks (LSTM) and deep belief networks (DBN) optimized with PSO - and a nonlinear weighted combination method based on PSO-DBN. The VMD technique initially decomposes the original wind power series to extract local features. Then, LSTM and PSO-DBN are used to build prediction models for the sub-series. Finally, these multiple sub-series forecasting models are integrated by a nonlinear weighted combination method based on PSO-DBN to form a hybrid model for short-term wind power forecasting. The study results demonstrate that the proposed method outperforms existing methods in effectiveness (Duan et al., 2022).

A research paper introduced a new hybrid model to predict carbon prices. The model is based on various methods such as Improved Complete Ensemble Empirical Mode Decomposition with Adaptive Noise (ICEEMDAN), Multiscale Fuzzy Entropy (MFE), Complete Ensemble Empirical Mode Decomposition (CEEMD), Improved Random Forest by Salp Swarm Algorithm (SSARF), Improved Backpropagation by Cuckoo Search (CSBP), Improved Extreme Learning Machine by Whale Optimization Algorithm (WOAELM), and Error Correction (EC). The proposed model outperformed other commonly used international carbon financial price forecasting models regarding forecast accuracy, with lower MAPE, MAE, and RMSE values. The results suggest that this model can provide a theoretical and data basis for carbon pricing and help formulate carbon reduction policies in China (Yang et al., 2023).

A study proposed an approach to cryptocurrency price prediction and portfolio allocation by considering it an MCDM problem. The researchers argued that the traditional time series forecasting approach is insufficient to tackle this field's dynamic challenges. The study employed the Prophet Forecasting Model (PFM) for time series forecasting to address this issue. The study extended Cluster analysis for asset allocation to improve the Multiple Criteria Decision Analysis (CLUS-MCDA) algorithm. The new algorithm incorporated additional features such as Density-Based Spatial Clustering of Applications with Noise (DBSCAN) and the Višekriterijumsko kompromisno rangiranje (VIKOR) methods. This approach provides a predictive and analytical framework that guides investors in making informed decisions (Maghsoodi, 2023).

The study's authors stated that accurately predicting water quality parameters is challenging due to water quality data's non-smooth and non-linear nature. Additionally, they noted that water quality parameters have a strong coupling relationship, further complicating the prediction process. The authors combined machine learning and intelligent optimization algorithms to overcome these challenges. They established a Back Propagation Neural Network (BPNN) model using the Artificial Bee Colony (ABC) algorithm. They incorporated three adaptive evolutionary strategies - dynamic adaptive factors, probability selection, and gradient initialization - to form the Adaptive Evolutionary Artificial Bee Colony (AEABC) algorithm (Chen et al., 2023).

A group of researchers have developed self-normalization (SN) to detect and estimate a single change point in the linear trend of a nonstationary time series. They have also combined the SN-based change-point test with the NOT algorithm to estimate multiple change points. The researchers used this technique to analyze the trajectory of the cumulative COVID-19 cases and deaths in 30 major countries. They have found some interesting patterns that could affect different countries' effectiveness of pandemic responses. Additionally, the researchers have designed a simple two-stage forecasting scheme for COVID-19 based on the change-point detection algorithm and a flexible extrapolation function. They have demonstrated its promising performance in predicting cumulative deaths in the United States (Jiang et al., 2023).

An academic paper used two methods to estimate coal prices: a time series method and a radial basis function (RBF) neural network method. To simulate the time series method, the authors used Monte Carlo simulation. Combining the two approaches could approximate coal prices with acceptable precision. The results showed that the combination of Brownian motion with mean return (BMMR) and RBF NN model (CBRN) was effective in reducing errors caused by various factors, such as economic and political conditions and differences in data (Sohrabi et al., 2023).

Researchers have pointed out that while conventional statistical and machine learning methods have been extensively used for forecasting, these methods have limitations in identifying nonlinear relations and modeling sequential data. To address this issue, these researchers employed a deep learning algorithm to predict air pollutants and meteorological features such as wind speed, wind direction, and air temperature in the Istanbul metropolitan area. To evaluate the efficacy of the GA approach, the researchers compared the prediction results of deep learning algorithms with default hyperparameters and random search algorithms. The proposed method outperformed the other configurations, with the mean squared error (MSE) reduced by 13.38% and 55.30% for testing performance, respectively. The experimental results indicated that GA is promising and applicable in hyperparameter optimization of deep neural network models (Erden, 2023).

It has been argued that time series analysis is a complex task due to the influence of various external factors like economic fluctuations and weather conditions. Due to this, accurate predictions are difficult to make. A machine-learning-based technique has been proposed to improve the precision and computation performance of time series forecasting models. This technique involves a multi-population PSO-based nonlinear time series predictive model that breaks the task into three sub-tasks. These sub-tasks are observation window optimization, predictive model induction task, and forecasting horizon prediction. The results showed that the proposed technique effectively induced a forecasting model with improved predictive and computation performance. Compared to benchmark techniques, the proposed technique performed better on all datasets (Kuranga et al., 2023).

A research paper introduced a computational forecasting model that simplifies determining the appropriate interval length and order of FTS—the model utilized PSO to identify the optimal interval length for partitioning the universe of discourse. Additionally, a dynamic order

approach was implemented to choose the order of FTS in the proposed model. During the training phase, a sequence of orders was established based on forecast accuracy, which was then utilized for forecasting by specific rules. The model was tested on various actual time series, including the benchmark data set of enrolments of Alabama University, the Taiwan stock exchange capitalization-weighted stock index, and West Texas Intermediate crude oil prices. The experimental results demonstrated that the proposed model outperforms existing models regarding forecasting accuracy (Goyal and Bisht, 2023).

In this study, we use numerical examples that have been conducted in previous studies. Song & Chissom (Song and Chissom, 1994) used FTS and its models for the first time in 1993. In general, the difference between the crisp time series and the FTS introduced by them is that the data value in the crisp ones is a number, while in the fuzzy ones, these values are in the form of linguistic values or fuzzy sets. Following this, they predicted the number of enrollments at the University of Alabama based on 20-year data using a time-independent model. Again, Song & Chissom (Song and Chissom, 1993) 1994 used the University of Alabama enrollment prediction based on a time-independent model and three-layer neural network method to de-fuzzify the output, lowering the average error.

In 1996, Chen et al. (Chen, 1996) modified Song and Chisom's method and claimed to have developed a better strategy than Song and Chisom's method. In 2001, Hurang (Huarng, 2001a) proposed an approach to improve prediction by integrating problem-specific heuristic knowledge and Chen's model. The predictive ability of these early studies on FTS drew the attention of researchers to this field. As a result, FTSs have been used as a powerful forecasting tool in a wide range of domains.

In 2007, Liu (2007) introduced an improved FTS model by considering the trapezoidal fuzzy number for the predicted value. In 2008, Chen et al. (CHENG et al., 2008) integrated the FCM clustering algorithm with FTS to effectively segment time series datasets. In 2011, Qiu et al. (2011) proposed a weighting method for fuzzy relations in the FTS modeling approach, which can improve prediction accuracy. In 2013, Singh and Borah (2013) combined a new data discretization approach with the FTS modeling approach to solve the problem of determining the length of effective intervals. Similarly, in 2018, Gupta et al. (2018) proposed a new hybrid model by integrating an automatic clustering approach with an FTS modeling approach. Bas et al. (2018) proposed applying an artificial neural network to determine fuzzy relations in the FTS modeling approach. In 2019, Chen et al. (2019) offered an FTS based on distance ratios and PSO technique to improve the forecasting ability of FTS. In 2020, Gao and Duru (2020) introduced a parsimonious time series, and Vovan and Lethithu (2022) proposed an FTS model based on improved fuzzy functions.

Studies show that the prediction accuracy of the FTS modeling approach mainly depends on two factors (Singh, 2017): determining the effective length of intervals and establishing fuzzy logic relationships (which are called decision rules). However, determining effective decision rules relies more on the appropriate selection of intervals. In the FTS modeling approach, researchers use different optimization algorithms to solve the problem of choosing the proper distances. For the right choice of distances, researchers mainly use GA (Chen and Chung, 2006), PSO (Kuo et al., 2010), Simulated Annealing (SA) (Lee et al., 2008), Geese Movement Based Optimization Algorithm (GMBOA) (Singh and Dhiman, 2018), Harmony Search Algorithm (Jiang et al., 2017) and QOA (Singh et al., 2018).

Time series data sets usually contain insufficient or incomplete information. In the case of the fuzzy-based modeling approach (i.e., the FTS modeling approach), the inherited uncertainty in the time series data set is characterized only by using the membership function. Intuitionistic Fuzzy Sets (IFS) theory has been introduced (Atanassov, 1986) to overcome this problem. In IFS, this lack of information is represented by both membership and non-membership functions.

This concept can help display time series data sets regarding membership and non-membership functions. Considering that many phenomena in the real world have a true part, a false part, and a part that cannot be commented on, Smarandache (1998) presented the concept of neutrosophic sets from a philosophical point of view, which can represent uncertainties in terms of determining the three degrees of membership, namely true function (Tf), indeterminate function (If) and falsehood function (Ff). A recent application of NS theory can be found in the classification of stock index datasets in terms of Tf, If, and Ff (Singh and Rabadiya, 2018). Except for this work, no other work has been reported in the literature that can provide a prediction model for time series data sets exclusively based on the NS theory. Accordingly, we were motivated to share our research on developing a time series prediction model utilizing NS theory and optimization algorithms.

3. Methodology and implementation

In this section, the proposed model for forecasting time series is introduced. Our proposed model is a combination of neutrosophic and optimization algorithms. The optimization algorithms we use are the QOA, GA, and PSO algorithms. First, the experiment uses the University of Alabama enrollment data set. Each step of the proposed model is explained below.

3.1. Algorithm 1 (NTS-QOA)

The proposed NTS-QOA algorithm for forecasting time series is as follows.

1 Gathering input data from the databases
2 Determining the number of intervals (n) and dividing the data in the database by n
3. Formulating the optimization function and specifying the objective function
4. Setting parameters related to the QOA optimization algorithm
5. Implementing the optimization algorithm on the data to minimize the prediction error
6. Updating the set of new intervals to improve the forecasting process
7 Converting crisp data to neutrosophic data
8 Calculating Entropy on the neutrosophic data set
9 Calculating the Neutrosophic Entropy Relationship (NER) parameter based on the entropy values obtained from the neutrosophic data set
10 Calculating Neutrosophic Entropy Relationship Group (NERG) based on the obtained NER values
11. Implementing the deneutrosophication operator to obtain predicted time series values
12. Evaluating the efficiency of the proposed model

To explain Algorithm 1, consider Table 1.

Let E_{min} and E_{max} be the minimum and maximum values of the time series data set. According to E_{min}, in the Universe of discourse, U_0 can be

Table 1
The university of Alabama enrollment dataset (Song and Chissom, 1994).

Row	Year	Enrollment	Row	Year	Enrollment
1	1971	13,055	12	1982	15,433
2	1972	13,563	13	1983	15,497
3	1973	13,867	14	1984	15,145
4	1974	14,696	15	1985	15,163
5	1975	15,460	16	1986	15,984
6	1976	15,311	17	1987	16,859
7	1977	15,603	18	1988	18,150
8	1978	15,861	19	1989	18,970
9	1979	16,807	20	1990	19,328
10	1980	16,919	21	1991	19,337
11	1981	16,388	22	1992	18,876

defined as $U_0 = [E_{min}\text{-}A_N, E_{max} + A_P]$, where A_N and A_P are two adjustment factors.

From Table 3, we have $E_{min} = 13{,}055$ and $E_{max} = 19{,}337$. At first, we assume $A_N = 3555$ and $A_P = 10{,}66$. Now the universe set U_0 is defined as $U_0 =$ (Zadeh, 1965), where min (U_0) = 9500 and max (U_0) = 30,000.

In the next step, to divide the reference set U_0 into n equal lengths of intervals, the following equation is used:

$$\mathbf{a_i} = \left[\mathbf{min(U_0)} + \mathbf{(i\text{-}1)}\frac{\mathbf{max(U_0)\text{-}min(U_0)}}{\mathbf{j}}, \mathbf{min(U_0)} + \mathbf{i}\frac{\mathbf{max(U_0)\text{-}min(U_0)}}{\mathbf{j}}\right] \tag{1}$$

for i = 1, 2, …, n, and j is the number of intervals.

Using Eqn. (1), the set of intervals resulting from the reference set U_0 can be defined as follows:

$$a_1 = [L_{B1}, U_{B1}], a_2 = [L_{B2}, U_{B2}], \ldots, a_n = [L_{Bn}, U_{Bn}]$$

where, L_{Bi}, $U_{Bi} \in a_i$ and $a_i \in U$.

Assuming j = 5 in Eqn. (1), the reference set $U_0 =$ (Zadeh, 1965) equal to 5 lengths of the reference set as $a_1 = [9500, 13{,}600]$, $a_2 = [13{,}600, 17{,}700]$, …, $a_5 =$ (Ihsan et al., 2023; Zadeh, 1965) is divided. In this set of intervals, we can assume $U_{B1} = 13{,}600 \in a_1$, $U_{B2} = 17{,}700 \in a_2$, …, $U_{B5} = 30{,}000 \in a_5$.

An optimization problem can be formulated based on the objective function for the third step. As the primary goal of this study is to improve prediction accuracy by selecting optimal intervals from the reference, set U_0, the model's performance should be determined by adopting the appropriate evaluation parameter. This parameter can also be used as an objective function. In the literature, the Average Forecasting Error Rate (AFER) is chosen by most researchers (Singh, 2016). Clearly, if the proposed model chooses optimal intervals, it will minimize AFER. Now, based on AFER, we can formulate the objective function as follows:

$$\mathbf{Minimize\ AFER} = \frac{\sum_{i=1}^{N}|\mathbf{A_i\text{-}F_i}|\mathbf{A_i}}{\mathbf{N}} \tag{2}$$

s.to

$$\begin{aligned}
&\mathbf{U_{B1}} \le \mathbf{max}\ (\mathbf{U_0}) \\
&\mathbf{U_{B2}} \le \mathbf{max}\ (\mathbf{U_0}) \\
&\quad\ldots \\
&\mathbf{U_{Bn}} \le \mathbf{max}\ (\mathbf{U_0}) \\
&\mathbf{U_{B1}} < \mathbf{U_{B2}} < \mathbf{U_{B3}} < \ldots < \mathbf{U_{Bn}}
\end{aligned}$$

where each F_i and A_i represent, respectively, the predicted and actual value of a particular year/day i and N is the total number of predicted years/days.

We have the set of upper bounds $U_{B1} = 13{,}600$, $U_{B2} = 17{,}700$, …, $U_{B7} = 30{,}000$. In terms of the set of upper bounds, the following constraints to minimize AFER - as defined in Eqn. (2) - are defined as follows:

$$\begin{aligned}
&13{,}600 \le 30{,}000 \\
&17{,}700 \le 30{,}000 \\
&\quad\vdots \\
&30{,}000 \le 30{,}000 \\
&13{,}600 < 17{,}700 < 20{,}800 < \ldots < 30{,}000
\end{aligned}$$

In step 4, the sub-steps of using QOA to minimize AFER are as follows.

- Sorting n number of intervals in the two-dimensional search space

The upper bounds of the set of intervals $a_1 = [L_{B1}, U_{B1}]$, $a_2 = [L_{B2}, U_{B2}]$, …, $a_n = [L_{Bn}, U_{Bn}]$ that is, U_{B1}, U_{B2}, …, U_{Bn} are arranged in ascending order as $U_{B1} < U_{B2} < \ldots < U_{Bn}$. Before starting the iteration process, it is assumed that this set of upper bounds is the best set of upper bounds (i.e., the set of upper bounds that gives the minimum AFER value).

For the set $U_0 =$ (Zadeh, 1965), the set of intervals is defined as $a_1 = [9500, 13{,}600]$, $a_2 = [13{,}600, 17{,}700]$, …, $a_5 =$ (Ihsan et al., 2023; Zadeh, 1965). This set of ranges consists of upper bounds of 13,600, 17,700, …, and 30,000. These upper limits are now arranged in ascending order as 13,600 < 17,700 < … < 30,000. At first, it is assumed that this set of upper bounds creates the optimal result in terms of AFER.

- Initialization of the search factors q_i (i = 1, 2, 3, …, n) for each of the upper bounds of U_{Bi}

$$\begin{aligned}
&\mathbf{q_1} = |\boldsymbol{\varphi}| \times \mathbf{Q_{1j}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{h1}} \in \mathbf{U_{B1}} \\
&\mathbf{q_2} = |\boldsymbol{\varphi}| \times \mathbf{Q_{2j}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{h2}} \in \mathbf{U_{B2}} \\
&\quad\vdots \\
&\mathbf{q_n} = |\boldsymbol{\varphi}| \times \mathbf{Q_{nj}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{hn}} \in \mathbf{U_{Bn}}
\end{aligned} \tag{3}$$

For the set of upper bounds 13,600, 17,700, …, 30,000, search factors are initialized q_1, q_2, …q_5, respectively. In Eqn. (3), the values of wave functions Q_{ij} and Q_{hi} are assumed between the range of random numbers [0, 1]. The true value of $\varphi = a + bi$ can be obtained as $|\varphi| = \sqrt{a^2 + b^2}$, which is assumed to be $a \in [0, 1]$ and $b \in [0, 1]$. Based on these assumptions, the search factors q_1, q_2, …, q_5 can be defined according to Eqn. (4) as follows:

$$\begin{aligned}
&\mathbf{q_1} = |\boldsymbol{\varphi}| \times \mathbf{Q_{1j}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{h1}} \in \mathbf{13{,}600} \\
&\mathbf{q_2} = |\boldsymbol{\varphi}| \times \mathbf{Q_{2j}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{h2}} \in \mathbf{17{,}700} \\
&\quad\vdots \\
&\mathbf{q_5} = |\boldsymbol{\varphi}| \times \mathbf{Q_{5j}} + (\mathbf{1}\text{-}|\boldsymbol{\varphi}|) \times \mathbf{Q_{h5}} \in \mathbf{30{,}000}
\end{aligned} \tag{4}$$

For Eqn. (4), the search factors and their corresponding calculation values are shown in Table 2.

- Obtaining the current position of each search agent q_i in terms of position $\vec{P}(x_i)$.

$$\vec{\mathbf{P}}(\mathbf{x_1}) = \frac{\mathbf{1}}{\mathbf{L_1}}\mathbf{e}^{-2/L_1} \in \mathbf{q_1}\ \vec{\mathbf{P}}(\mathbf{x_2}) = \frac{\mathbf{1}}{\mathbf{L_2}}\mathbf{e}^{-2/L_2} \in \mathbf{q_2} : \vec{\mathbf{P}}(\mathbf{x_n}) = \frac{\mathbf{1}}{\mathbf{L_n}}\mathbf{e}^{-2/L_n} \in \mathbf{q_n} \tag{5}$$

Here, each $\vec{P}(x_i)$ and L_i represent, respectively, the current position and search domain of each search agent q_i, where i = 1, 2, 3, …, n.

According to Eqn. (5), the current positions of search agents q_1‘ q_2‘ … ‘ q_5 can be calculated as follows:

$$\begin{aligned}
\vec{P}(x_1) &= \frac{1}{L_1}e^{-2/L_1} = \frac{1}{13{,}600}e^{-2/13{,}600} = 74 \times 10^{-6} \in q_1\ \vec{P}(x_2) = \frac{1}{L_2}e^{-2/L_2} \\
&= \frac{1}{17{,}700}e^{-2/17{,}700} = 56 \times 10^{-6} \in q_2 : \vec{P}(x_5) = \frac{1}{L_5}e^{-2/L_5} = \frac{1}{30{,}000}e^{-2/30{,}000} \\
&= 33 \times 10^{-6} \in q_5
\end{aligned} \tag{6}$$

Table 2
Search factors and their corresponding calculation values.

Search factors	a	b	$\lvert\varphi\rvert = \sqrt{a^2 + b^2}$	Q_{ij}	Q_{hi}	Final value
q_1	0.5	0.37	0.65	0.06	0.06	0.06
q_2	0.7	0.49	10.9	0.53	0.53	0.53
q_3	0.8	0.16	0.89	0.98	0.96	0.98
q_4	0.0	0.90	0.90	0.77	0.01	0.70
q_5	0.5	0.30	0.60	0.18	0.59	0.34

Table 3
NTS display from the University of Alabama Enrollment dataset.

Year	Enrollment	NTS	Year	Enrollment	NTS
1971	13,055	$\frac{13055}{< 0.2, 08, 0.82 >}$	1982	15,603	$\frac{15433}{< 0.1244, 0.8756, 0.8844 >}$
1972	13,563	$\frac{13563}{< 0.0852, 0.9148, 0.9188 >}$	1983	15,861	$\frac{15497}{< 0.1258, 0.8742, 0.8832 >}$
1973	13,867	$\frac{13867}{< 0.0916, 0.9084, 0.913 >}$	1984	16,807	$\frac{15145}{< 0.1184, 0.8816, 0.8895 >}$
1974	14,696	$\frac{14696}{< 0.109, 0.891, 0.8977 >}$	1985	16,919	$\frac{15163}{< 0.1188, 0.8812, 0.8892 >}$
1975	15,460	$\frac{15460}{< 0.125, 0.875, 0.8839 >}$	1986	15,497	$\frac{15984}{< 0.136, 0.864, 0.8747 >}$
1976	15,311	$\frac{15311}{< 0.1219, 0.8781, 0.8866 >}$	1987	18,970	$\frac{16859}{< 0.1543, 0.8457, 0.8596 >}$
1977	16,388	$\frac{15603}{< 0.128, 0.872, 0.8814 >}$	1988	19,328	$\frac{18150}{< 0.1814, 0.8186, 0.8385 >}$
1978	15,433	$\frac{15861}{< 0.1334, 0.8666, 0.8768 >}$	1989	19,337	$\frac{18970}{< 0.1986, 0.8014, 0.8256 >}$
1979	15,984	$\frac{16807}{< 0.1532, 0.8468, 0.8605 >}$	1990	15,145	$\frac{19328}{< 0.2061, 0.7939, 0.8202 >}$
1980	16,859	$\frac{16919}{< 0.1556, 0.8444, 0.8586 >}$	1991	15,163	$\frac{19337}{< 0.2063, 0.7937, 0.8201 >}$
1981	18,150	$\frac{16388}{< 0.1444, 0.8556, 0.8677 >}$	1992	18,876	$\frac{18876}{< 0.1966, 0.8034, 0.8271 >}$

In Eqn. (5), each search range L_i shows the current position of the search agent q_i. Therefore, for the search domains L_1‘ L_2‘ … ‘ L_5, we have considered the initial values of the upper bounds U_{B1}‘ U_{B2}‘ … ‘ U_{B5}, respectively, in Eqn. (6).

- Calculating the displacement of each search factor q_i between 0 and 1

$$\boldsymbol{D(q_1) = \int_0^1 \left|\overrightarrow{P}(x_1)\right| dx = e^{-2/L_1} D(q_2) = \int_0^1 \left|\overrightarrow{P}(x_2)\right| dx = e^{-2/L_2} \vdots D(q_n) = \int_0^1 \left|\overrightarrow{P}(x_n)\right| dx = e^{-2/L_n}} \tag{7}$$

The displacement of each search factor $q_1, q_2, \ldots, q_5$ can be calculated using Eqn. (7) as follows:

$$D(q_1) = \int_0^1 \left|\overrightarrow{P}(x_1)\right| dx = e^{-2/L_1} = e^{-2/13{,}600} = 0.99 D(q_2)$$

$$= \int_0^1 \left|\overrightarrow{P}(x_2)\right| dx = e^{-2/L_2} = e^{-2/17{,}700}$$

$$= 0.99 \vdots D(x_5) = \int_0^1 \left|\overrightarrow{P}(x_5)\right| dx = e^{-2/L_5}$$

$$= e^{-2/30{,}000} = 0.99$$

- Generating a random position y_i for each search factor q_i

$$\boldsymbol{y_1 = \left|q_1 \pm \frac{L_1}{2} ln\,(1/v)\right|, v \in [0,1]}$$
$$\boldsymbol{y_2 = \left|q_2 \pm \frac{L_2}{2} ln\,(1/v)\right|, v \in [0,1] \vdots} \tag{8}$$
$$\boldsymbol{y_n = \left|q_n \pm \frac{L_n}{2} ln\,(1/v)\right|, v \in [0,1]}$$

where v is a random number with a value between [0, 1].

The random position for each search factor $q_1, q_2, \ldots, q_5$ is created using Eqn. (8) as follows:

$$y_1 = q_1 - \frac{L_1}{2} ln(1/v) = 0.06 - \frac{13{,}600}{2} ln(1/0.65) = 1272.5$$

$$y_2 = q_2 - \frac{L_2}{2} ln(1/v) = 0.53 - \frac{17{,}700}{2} ln(1/0.73) = 1209 \vdots$$

$$y_5 = q_5 - \frac{L_5}{2} ln(1/v) = 0.34 - \frac{30{,}000}{2} ln(1/0.55) = 3894.6$$

Here, the operator "−" was used instead of "±" (see Eqn. (8)). The value of search factors $q_1, q_2, \ldots, q_5$ can be obtained from Table 2.

- Calculating the search range of each search agent q_i according to the upper limit $U_{Bi} \in U$ and the random position corresponding to y_i

$$\mathbf{L_1 = 2\bullet|U_{B1}\text{-}y_1| \in \overrightarrow{P}(x_1) L_2 = 2\bullet|U_{B2}\text{-}y_2| \in \overrightarrow{P}(x_2) \vdots L_n = 2\bullet|U_{Bn}\text{-}y_n| \in \overrightarrow{P}(x_n)} \tag{9}$$

For each search factor $q_1, q_2, \ldots, q_5$, the search domain corresponding to them, that is, $L_1, L_2 \ldots, L_5$, can be calculated using Eqn. (9) as follows:

$$L_1 = 2\bullet|U_{B1}\text{-}y_1| = 2\bullet|13{,}600\text{-}1272.5| = 24{,}655 \in \overrightarrow{P}(x_1) L_2$$

$$= 2 \bullet |U_{B2}\text{-}y_2| = 2\bullet|17{,}700\text{-}1209| = 32{,}982 \in \overrightarrow{P}(x_2) \vdots L_5$$

$$= 2 \bullet |U_{B5}\text{-}y_5| \in \overrightarrow{P}(x_5) = 2\bullet|30{,}000\text{-}3894.6| = 52{,}210.8$$

$$\in \overrightarrow{P}(x_5)$$

- Obtaining the final position of each search factor q_i

$$\boldsymbol{y_{f1} = q_1 \pm \alpha\bullet|U_{B1}\text{-}y_1| \bullet ln(1/v)}$$
$$\boldsymbol{y_{f2} = q_2 \pm \alpha\bullet|U_{B2}\text{-}y_2| \bullet ln(1/v) \vdots} \tag{10}$$
$$\boldsymbol{y_{fn} = q_n \pm \alpha\bullet|U_{Bn}\text{-}y_n| \bullet ln\,(1/v)}$$

Here, each y_{fi} represents the corresponding final position for the search factor q_i, and α represents the tuning parameter of the algorithm.

The final position for each search factor $q_1, q_2, \ldots, q_5$ is calculated using Eqn. (10) as follows:

$$y_{f1} = q_1\text{-}\alpha\bullet|U_{B1}\text{-}y_1| \bullet ln(1/v) = 0.36\text{-}0.3\cdot|13{,}600\text{-}1272.5| ln(1/0.65) = \text{-}691.53$$
$$y_{f2} = q_2\text{-}\alpha\bullet|U_{B2}\text{-}y_2| \bullet ln(1/v) = 0.26\text{-}0.3\cdot|17{,}700\text{-}1272.5| ln(1/0.73) = \text{-}673.32 \vdots$$
$$y_{f5} = q_5\text{-}\alpha\bullet|U_{B5}\text{-}y_5| \bullet ln(1/v) = 0.06\text{-}0.3\cdot|30{,}000\text{-}3894.6| ln(1/0.55) = \text{-}2237.9$$

Here, the operator "−" is used instead of "±" and the value of 0.3 is

considered for α.

- Updating each search factor q_i according to the upper bound $U_{Bi} \in U$ and the previous corresponding position $\vec{P}(x_1)$.

$$\vec{\boldsymbol{P}}(\boldsymbol{x_1}+\mathbf{1})=\vec{\boldsymbol{P}}(\boldsymbol{x_1})+\boldsymbol{y_{f1}}\in\boldsymbol{x_1}\vec{\boldsymbol{P}}(\boldsymbol{x_2}+\mathbf{1})=\vec{\boldsymbol{P}}(\boldsymbol{x_2})+\boldsymbol{y_{f2}} \in \boldsymbol{x_2}:\vec{\boldsymbol{P}}(\boldsymbol{x_n}+\mathbf{1})=\vec{\boldsymbol{P}}(\boldsymbol{x_n})+\boldsymbol{y_{fn}}\in\boldsymbol{x_n} \tag{11}$$

Here $\vec{P}(x_i+1)$ and $\vec{P}(x_i)$ represent the new and previous positions of each search factor q_i in the search space, respectively.

For each search factor $q_1, q_2, \ldots, q_5$, the position is updated based on Eqn. (11):

$$\vec{P}(x_1+1)=\vec{P}(x_1)y_{f1}=13,600\text{-}691.53=12,908.47$$

$$\in x_1\vec{P}(x_2+1)=\vec{P}(x_2)+y_{f2}=17,700\text{-}673.32$$

$$=17,026.68\in x_2:\vec{P}(x_5+1)=\vec{P}(x_5)+y_{f5}$$

$$=30,000\text{-}2273.9=27,762.1\in x_5$$

In the next step, Updating the set of new intervals to improve the forecasting process is done as follows:

The set of new intervals can be defined as follows:

$$a_1(\text{new})=\left|L_{B1}\,\vec{P}(x_1+1)\right|,a_2(\text{new}) =\left|\vec{P}(x_1+1),\vec{P}(x_2+1)\right|,\ldots,a_n(\text{new})=\left|\vec{P}(x_{n-z}+1),\vec{P}(x_n+1)\right|$$

where the new reference set U_{1st} can be defined as $U_{1st}=|\vec{P}(x_n+1)|$. Here, L_{B1} and $\vec{P}(x_n+1)$ represent the lower and upper bounds of the new reference set U_{1st}, respectively. It should be noted that the reference set obtained after the ith iteration has a U_i representation, where $i = 1, 2, 3, \ldots, n$.

The new set of intervals can be defined as follows:

$$a_1(\text{new})=[9500,12,325.34],a_2(\text{new})=[12,908.47,16432.13],\ldots,a_5(\text{new}) =[12,338.1,27,002.11]$$

For this new set of intervals, their reference set can be defined as $U_{1st}=[9500,27002.11]$.

In step 7, crisp data is converted to neutrosophic data.

The time series data set in the NTS data set can be displayed as follows:

$$\mathbb{N}_{t_i}=\frac{\mathbf{t_i}}{<\mathbf{T_f}(\mathbf{t_i}),\mathbf{I_f}(\mathbf{t_i}),\mathbf{F_f}(\mathbf{t_i})>} \tag{12}$$

where $T_f, I_f, F_f : U_{1st}\to[0,1]$ and $\forall t_i \in U_{1st}$, $t_i \equiv t_i(T_f(t_i), I_f(t_i), F_f(t_i))t_i))$ $T_f(t_i), I_f(t_i), F_f \in \mathbb{N}_{t_i}$.

The values of $T_f(t_i)\text{‹}I_f(t_i)\text{and}F_f(t_i)$ can be defined according to the new discourse $U_{1st}=[L_{B1}\,\vec{P}(x_n+1)]$ using the following equations:

$$\mathbf{T_f}(\mathbf{t_i})=\frac{\mathbf{t_i}\text{-}\mathbf{L_{B1}}}{\vec{\mathbf{P}}(\mathbf{x_n}+\mathbf{1})\text{-}\mathbf{L_{B1}}} \tag{13}$$

$$\mathbf{F_f}(\mathbf{t_i})=\mathbf{1}\text{-}\mathbf{T_f}(\mathbf{t_i}) \tag{14}$$

$$\mathbf{I_f}(\mathbf{t_i})=\sqrt{\mathbf{T_f}(\mathbf{t_i})^2+\mathbf{F_f}(\mathbf{t_i})^2} \tag{15}$$

For example, consider the enrollment value for 1971, which is 13,055. Now, neutrosophication can be done using Eqns. 13–15 as follows:

$$T_f(13,055)=\frac{t_i\text{-}L_B}{U_B(\text{new})\text{-}L_B}=\frac{13,055\text{-}9500}{27,002.11\text{-}9500}=0.2$$

$$F_f(13,055)=1\text{-}T_f(t_i)=1\text{-}0.2=0.8$$

$$I_f(13,055)=\sqrt{T_f(t_i)^2+F_f(t_i)^2}=\sqrt{0.2^2+0.8^2}=0.82$$

Now based on $T_f(13,055)$, $F_f(13,055)$, and $I_f(13,055)$, the registration value of "13,055″ in NTS is displayed as follows:

$$\mathbb{N}_{13,055}=\frac{t_i}{<T_f(t_i),I_f(t_i),F_f(t_i)>}=\frac{13,055}{<0.2,0.8,0.82>}$$

A complete representation of the University of Alabama enrollment dataset in the NTS dataset is shown in Table 3.

In the next step, the entropy is calculated on the neutrosophic data set as follows:

$$\mathbf{E}_{\mathbb{N}}(\mathbf{t_i})=\mathbf{1}\text{-}\frac{\mathbf{1}}{\mathbf{n}}\sum_{\mathbf{t_i}\in\mathbf{U_{1st}}}\left(\mathbf{T_f}(\mathbf{t_i})+\mathbf{I_f}(\mathbf{t_i})+\mathbf{F_f}(\mathbf{t_i})\right)\times\mathbf{E_1E_2E_3} \tag{16}$$

where $E_1=|T_f(t_i)\text{-}T_f^c(t_i)|\text{‹}E_2=|I_f(t_i)\text{-}I_f^c(t_i)|$ and $E_3=|F_f(t_i)\text{-}F_f^c(t_i)|$, and $U_{1st}=|L_{B1},\vec{P}(x_n+1)|$ is a new reference set.

For example, consider the NTS display for the register value "13,055." Now, its corresponding entropy value can be obtained using Eqn. (16):

$$E_{\mathbb{N}}(13,055)=1\text{-}\frac{1}{3}\sum_{t_i\in a_i(\text{new})}(0.2+0.8+0.83)\times0.6\times0.64\times0.6=0.6$$

where $E_1 = |0.2\text{-}0.8| = 0.6$, $E_2 = |0.82\text{-}0.18| = 0.64$, and $E_3 = |0.8\text{-}0.2| = 0.6$.

Thus, the entropy for the NTS data set can be given by the new reference set $U_{1st}=|L_{B1},\vec{P}(x_n+1)|$, as shown in Table 4.

In the next step, the NER parameter is calculated based on the entropy values obtained from the neutrosophic data set.

NER can be established between two consecutive NTS data set entropy values. A NER represents the relationship between two or more consecutive entropies. For example, $E_{\mathbb{N}}(t_i)$ and $E_{\mathbb{N}}(t_{i+1})$ are two consecutive entropy values in the NTS dataset. NER between them can be set as $E_{\mathbb{N}}(t_i)\to E_{\mathbb{N}}(t_{i+1})$.

In Table 4, the entropy values for 1973 and 1974 are 0.67 and 0.71, respectively. So, we can set a NER between 0.67 and 0.71 as 0.67 → 0.71. Thus, we have obtained the NERs for the NTS data set of University Enrollment, shown in Table 5.

In this step, the NERG is created among NERs:

NERGs can be created between two or more NERs. In the set of NERs, relations with the same previous state can be grouped, called NERGs.

Here is an example of creating a NERG from the list of NERs in Table 5. In this table, two NERs have the same status as before: 0.71 → 0.73 and 0.71 → 0.72. NERG can be formed among these NERs as 0.71 → 0.72, 0.73. We have thus generated NERGs for the University of Alabama enrollment data set presented in Table 6.

Table 4
Entropy for NTS dataset.

Year	Entropy	Year	Entropy
1971	0.615	1982	0.7225
1972	0.64	1983	0.7225
1973	0.67	1984	0.715
1974	0.71	1985	0.7367
1975	0.7225	1986	0.7733
1976	0.7367	1987	0.7967
1977	0.745	1988	0.845
1978	0.7733	1989	0.865
1979	0.785	1990	0.8633
1980	0.7967	1991	0.8633
1981	0.75	1992	0.85

In the next step, the de-fuzzification operation is performed to obtain the predicted time series values according to the following stages.

- Getting the corresponding entropy value for a specific day\year t_i as $E_{\mathbb{N}}(t_i)$.
- Finding the NERG for the corresponding $E_{\mathbb{N}}(t_i)$, which can be represented as:

$$\mathbf{E}_{\mathbb{N}}(\mathbf{t_i}) \rightarrow \mathbf{E}_{\mathbb{N}}(\mathbf{t_{k1}}) \rightarrow \mathbf{E}_{\mathbb{N}}(\mathbf{t_{k2}}) \rightarrow \mathbf{E}_{\mathbb{N}}(\mathbf{t_{k3}}), \ldots, \mathbf{E}_{\mathbb{N}}(\mathbf{t_{kn}}) \tag{17}$$

where $E_{\mathbb{N}}(t_i)$ is the previous state value of the NTS for day/year t_i, while $E_{\mathbb{N}}(t_{k1})$, $E_{\mathbb{N}}(t_{k2})$, …, $E_{\mathbb{N}}(t_{kn})$ respectively, are called the current state of NTS values for days\years $t_{k1}‘t_{k2}‘t_{k3}‘ \ldots ‘ t_{kn}$.

- Obtaining the optimal entropy available in the current state of NERG

$$\mathbf{E}_{\mathbb{N}}(\mathbf{optimal}) = \frac{\mathbf{1}}{\mathbf{n}} \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{E}_{\mathbb{N}}(\mathbf{t_{kn}}) \tag{18}$$

- Obtaining the real values of the time series corresponding to the previous state of NERG as t_i
- Applying the following de-fuzzification formula to calculate the predicted value for a day\year t_{i+1}

$$\mathbf{Forecasting}\ (\mathbf{t_{i+1}}) = \frac{\mathbf{E}_{\mathbb{N}}(\mathbf{optimal})}{\mathbf{E}_{\mathbb{N}}(\mathbf{t_i})} \tag{19}$$

Suppose we want to predict the registration of the year “1976". For this year, the corresponding entropy value is 0.73. NERG is then obtained for an entropy value of 0.73 as 0.73, 0.74, 0.77, where 0.73 is the previous state, and 0.74, 0.77 is the current NERG state. Now we have:

$$E_{\mathbb{N}}(\text{optimal}) = \frac{0.74 + 0.77}{2} = 0.75$$

$$\text{Forecasting}(1976) = \frac{0.75 \times 15,311}{0.73} = 15,730.4$$

The proposed model’s efficiency is evaluated in the algorithm’s last step.

Efficiency is evaluated using AFER, Eqn. (2). If the AFER value shows that there is more scope for improving the performance of the proposed model, then the procedure of the previous sub-steps is repeated.

After a certain number of repetitions, the calculation is stopped if the desired result is obtained. This study repeats the proposed NTS-QOA model 100 times for optimal solutions. The reference set and corresponding intervals that create the optimal AFER can be considered the best global solution.

Before applying QOA in the proposed NTS-QOA model, an initial assumption has been made for the reference set as $U_0 =$ (Zadeh, 1965) for the enrollment data set of the University of Alabama. In step 2 of the proposed model, $U_0 =$ (Zadeh, 1965) is divided into five equal lengths of intervals as $a_1 = [9500, 13,600]$, $a_2 = [13,600, 17,700]$, $a_3 = [17,700, 21,800]$, $a_4 = [21,800, 25,900]$, and $a_5 =$ (Ihsan et al., 2023; Zadeh, 1965).

Table 5
NERs for the University of Alabama enrollment dataset.

NERs	
0.61 → 0.64	0.72 → 0.71
0.73 → 0.74	0.84 → 0.86
0.75 → 0.72	0.71 → 0.72
0.77 → 0.79	0.78 → 0.79
0.86 → 0.85	0.71 → 0.73
0.64 → 0.67	0.86 → 0.86
0.74 → 0.77	0.72 → 0.73
0.72 → 0.72	0.79 → 0.75
0.79 → 0.84	0.73 → 0.77
0.67 → 0.71	0.86 → 0.86
0.77 → 0.78	–

Table 6
NERGs for the dataset.

NERGs
0.6 → 0.64
0.67 → 0.71
0.71 → 0.72, 0.73
0.72 → 0.73, 0.72
0.73 → 0.74, 0.77
0.74 → 0.77
0.77 → 0.78, 0.79
0.78 → 0.79
0.75 → 0.72

In the search for the optimal reference set and its corresponding intervals, the upper bounds, displayed as $U_{B1} = 13,600 \in a_1$, $U_{B2} = 17,700 \in a_2$, $U_{B3} = 21,800 \in a_3$, $U_{B4} = 25,900 \in a_4$ and $U_{B5} = 30,000 \in a_5$ in step 2, are considered. Now, each upper limit in the form of $13,600 \in q_1$, $17,700 \in q_2$, $21,800 \in q_3$, $25,900 \in q_4$ and $30,000 \in q_5$ is assigned to each search agent. The initial Random Position (RM) of each search factor $q_{i\ (i=1,\ 2,\ \ldots,\ n)}$ for the reference set is shown as a column in Table 7. For ease of calculation, the upper bound given to each search factor in this column specialization is arranged such that $U_{B1} < U_{B2} < U_{B3} < U_{B4} < U_{B5}$.

To find the optimal solution, these five search factors are allowed to move to other positions, and their movement is recorded and considered in the first iteration. During the movement of any search factor from one place to another, the upper bounds belonging to the new array must always be set such that they always consist of the ascending sequence of $U_{B1} < U_{B2} < U_{B3} < U_{B4} < U_{B5}$ follow.

For the first iteration, the position of each search factor is updated based on the reference set $U_0 =$ (Zadeh, 1965). Each search factor’s new position is considered their local best position (LB). These best local positions for the first iteration are shown in Table 7. Using the best local positions of the search factor, we define a new set of intervals and a new reference set. The first iteration’s new reference set is $U_{1st} =$ [9500, 27,762.10]. The intervals corresponding to this reference set are listed in Table 9. NERHs are made based on the U_{1st} of NERs. Finally, the first iteration ends with the de-fuzzification operation. The predicted university enrollment values obtained after the first iteration are shown in Table 10. After the first iteration, the proposed NTS-QOA model reaches an AFER of 0.167% (see Table 11).

For the 99th iteration, the timed position of each search factor is shown in Table 7. For iteration 99, the parameter values associated with the five search universes are listed in Table 8. Using the best local positions of the search factors, we define a new set of intervals and a new reference set. The new reference set resulting from iteration 99 is displayed as $U_{99th} =$ [9500, 25,579.6]. Based on the U_{99th}, the predicted university enrollment values are shown in Table 10. In this case, the proposed NTA-QOA model achieves an AFER of 0.167%.

Finally, after reaching 100 iterations, the computational process of the proposed NTS-QOA model is stopped. For the hundredth iteration, the timed position of each search factor is shown in Table 7. The parameter values corresponding to the five search factors are listed in Table 8. The new reference set resulting from the hundredth iteration is shown as $U_{100th} =$ [9500, 24,866.90]. The predicted university enrollment values after the hundredth iteration are shown in Table 10. After the hundredth iteration, the proposed NTS-QOA model reaches AFER of 0.166%.

Table 10 shows that the proposed NTS-QOA model reached AFERs of 0.167, 0.166, and 0.166, respectively, after the 1st, 99th, and 100th iterations. Hence, the predicted values of available university enrollment after the 100th iteration can be considered optimal. And the reference set and its corresponding intervals, through which these predicted values are obtained, can be regarded as the best global.

Table 7
The positions of each search factor before and after iterations.

Position	Iteration	Reference set	q_1	q_2	q_3	q_4	q_5
RM	0	U_0 = [9500, 30,000]	13,600	17,700	21,800	25,900	30,000
LB	1	U_{1st} = [9500, 27762.10]	12908.47	17026.68	20648.80	23338.10	27762.10
LB	99	U_{99th} = [9500, 25579.60]	13599.97	16682.80	18341.70	25233.42	25579.60
GB	100	U_{100th} = [9500, 24866.90]	13303.33	15054.50	21633.69	23019.50	24866.90

*The abbreviations RM, LB, and GB in the Position column stand for random, local best, and global best, respectively.

Table 8
Selected parameter values corresponding to 5 search factors (iterations 1, 99, and 100).

Iteration	Factor	a	b	$\lvert\varphi\rvert$	Q_{ij}	Q_{hi}	q_n	$\vec{P}(x_i)$	D (q_i)	v	y_i	L_i	α	y_{fi}	$\vec{P}(x_i+1)$
1	q_1	0.53	0.37	0.65	0.06	0.06	0.06	0.00	1.00	0.33	7570.43	12059.15	1.11	−2013.77	11586.23
	q_2	0.76	0.49	0.91	0.53	0.53	0.53	0.00	1.00	0.63	4098.65	27202.69	0.46	−1889.45	15810.55
	q_3	0.88	0.16	0.89	0.98	0.96	0.98	0.00	1.00	0.72	3567.18	36465.64	0.33	−1789.60	20010.40
	q_4	0.07	0.90	0.90	0.77	0.01	0.70	0.00	1.00	0.05	39202.96	26605.91	3.03	−12080.95	13819.05
	q_5	0.52	0.30	0.60	0.18	0.59	0.34	0.00	1.00	0.60	7694.11	44611.77	0.51	−3432.29	26567.71
99	q_1	0.20	0.24	0.31	0.62	0.62	0.62	0.00	1.00	0.63	3105.94	20988.11	0.46	−1437.64	12162.36
	q_2	0.14	0.72	0.73	0.40	0.46	0.42	0.00	1.00	0.91	878.97	33642.06	0.10	−501.01	17198.99
	q_3	0.71	0.40	0.81	0.01	0.07	0.03	0.00	1.00	0.63	5026.60	33546.80	0.46	−2320.53	19479.47
	q_4	0.59	0.45	0.74	0.93	0.09	0.71	0.00	1.00	0.01	55069.47	58338.95	4.25	−37212.46	11312.46
	q_5	0.38	0.55	0.66	0.11	0.90	0.38	0.00	1.00	0.32	17257.15	25485.69	1.15	−4397.82	25602.18
100	q_1	0.11	0.63	0.64	0.67	0.67	0.67	0.00	1.00	0.88	870.52	25458.97	0.13	−488.58	13111.42
	q_2	0.48	0.31	0.57	0.52	0.71	0.60	0.00	1.00	0.37	8711.95	17976.10	0.98	−2653.94	15046.06
	q_3	0.81	0.32	0.87	0.31	0.34	0.32	0.00	1.00	0.77	2894.84	37810.32	0.27	−1506.11	20293.89
	q_4	0.87	0.17	0.89	0.85	0.96	0.86	0.00	1.00	0.17	23092.56	5614.89	1.78	−1501.07	24398.93

Table 9
List of interval and reference sets corresponding to 5 search factors (Iterations 1, 99, and 100).

Iteration	a_1 (new)	a_2 (new)	a_3 (new)	a_4 (new)	a_5 (new)	Set
1	[9500, 12,908.47]	[12,908.47, 17,026.68]	[17,026.68, 20,648.80]	[20,648.80, 23,338.10]	[23,338.10, 27,762.10]	[9500, 27,762.10] ∈ U1st
99	[9500, 13,599.97]	[13,599.97, 16,682.80]	[16,682.80, 18,341.70]	[18,341.70, 25,233.42]	[25,233.42, 25,579.60]	[9500, 25,579.60] ∈ U_{99th}
100	[9500, 13,303.33]	[13,303.33, 15,054.50]	[15,054.50, 21,633.69]	[21,633.69, 23,019.50]	[23,019.50, 24,866.90]	[9500, 24,866.90] ∈ U100th

3.2. Algorithm 2 (NTS-GA)

The proposed NTS-GA model for forecasting time series.

1. Gathering input data from the desired databases
2. Determining the number of intervals (n) and dividing the data in the database by n
3. Setting parameters related to the GA optimization algorithm
4. Implementing the optimization algorithm on the data to minimize the prediction error
5. Updating the set of new intervals to improve the forecasting process
6 Converting crisp data to neutrosophic data
7 Calculating Entropy on the neutrosophic data set
8. Calculating the NER parameter based on the entropy values obtained from the neutrosophic data set
9. Calculating NERG based on the obtained NER values
10. Implementing deneutrosophicization operator to obtain predicted time series values
11. Evaluating the efficiency of the proposed model

In Algorithm 2, all the steps are followed as in Algorithm 1, with the difference that the genetic optimization algorithm is used to reduce the prediction error.

We divide the statistical population into sub-intervals ($n = 5$) and randomly select two parents from each sub-interval. We perform mutation and intersection operators on the parent differently to get new children from the above operators. Evaluator function (AFER amount obtained from new children): if the AFER result of the children is improved compared to the parent, we replace the parent with the obtained children. Otherwise, we will choose other parents. We continue this work until either the algorithm converges, that is, we reach the minimum value of AFER based on the desired value, or we terminate the algorithm's execution after several repetitions and no change in the mentioned parameter. The best result of AFER, its minimum value, is to save the desired effect and use the resulting parameters to predict the real data. Because the selections are local and random in the GA, the convergence result is less fast and accurate than the QOA and PSO algorithms.

3.3. Algorithm 3 (NTS-PSO)

The proposed NTS-PSO model for forecasting time series.

1 Gathering input data from the desired databases
2 Determining the number of intervals (n) and dividing the data in the database by n
3. Setting parameters related to the PSO optimization algorithm
4 Implementing the optimization algorithm on the data to minimize the prediction error
5 Updating the set of new intervals to improve the forecasting process
6. Converting crisp data to neutrosophic data
7. Calculating Entropy on the neutrosophic data set
8 Calculating the NER parameter based on the entropy values obtained from the neutrosophic data set
9 Calculating NERG based on the obtained NER values
10 Implementing deneutrosophicization operator to obtain predicted time series values
11 Evaluating the efficiency of the proposed model

Table 10
Predicted university enrollment values obtained after iterations 1, 99, and 100 using the proposed NTS-QOA model.

Iteration	Year	Real enrollment	Entropy	Optimal entropy	Predicted enrollment	AFER%
1	1971	13,055	0.86	0.88	13358.6	0.167402
	1972	13,563	0.9	0.905	13638.35	
	1973	13,867	0.91	0.93	14171.77	
	1974	14,696	0.95	0.96	14850.69	
	1975	15,460	0.97	0.97	15,460	
	1976	15,311	0.97	0.97	15,311	
	1977	15,603	0.97	0.97	15,603	
	1978	15,861	0.98	0.986667	15968.9	
	1979	16,807	0.99	0.988	16773.05	
	1980	16,919	0.99	0.988	16884.82	
	1981	16,388	0.99	0.988	16354.89	
	1982	15,433	0.97	0.97	15,433	
	1983	15,497	0.97	0.97	15,497	
	1984	15,145	0.96	0.966667	15250.17	
	1985	15,163	0.96	0.966667	15268.3	
	1986	15,984	0.98	0.986667	16092.73	
	1987	16,859	0.99	0.988	16824.94	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	
99	1971	13,055	0.82	0.84	13373.41	0.16658
	1972	13,563	0.86	0.87	13720.71	
	1973	13,867	0.88	0.9	14182.16	
	1974	14,696	0.92	0.935	14935.61	
	1975	15,460	0.95	0.948	15427.45	
	1976	15,311	0.94	0.9475	15433.16	
	1977	15,603	0.95	0.948	15570.15	
	1978	15,861	0.96	0.973333	16081.29	
	1979	16,807	0.98	0.9825	16849.88	
	1980	16,919	0.98	0.9825	16962.16	
	1981	16,388	0.97	0.96	16219.05	
	1982	15,433	0.95	0.948	15400.51	
	1983	15,497	0.95	0.948	15464.37	
	1984	15,145	0.94	0.9475	15265.84	
	1985	15,163	0.94	0.9475	15283.98	
	1986	15,984	0.96	0.973333	16,206	
	1987	16,859	0.98	0.9825	16902.01	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	
100	1971	13,055	0.82	0.84	13373.41	0.16658
	1972	13,563	0.86	0.87	13720.71	
	1973	13,867	0.88	0.9	14182.16	
	1974	14,696	0.92	0.935	14935.61	
	1975	15,460	0.95	0.948	15427.45	
	1976	15,311	0.94	0.9475	15433.16	
	1977	15,603	0.95	0.948	15570.15	
	1978	15,861	0.96	0.973333	16081.29	
	1979	16,807	0.98	0.9825	16849.88	
	1980	16,919	0.98	0.9825	16962.16	
	1981	16,388	0.97	0.96	16219.05	
	1982	15,433	0.95	0.948	15400.51	
	1983	15,497	0.95	0.948	15464.37	
	1984	15,145	0.94	0.9475	15265.84	
	1985	15,163	0.94	0.9475	15283.98	
	1986	15,984	0.96	0.973333	16,206	
	1987	16,859	0.98	0.9825	16902.01	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	

Generally, algorithms based on cumulative particle methods are similar to basic evolutionary algorithms such as genetics. In cumulative methods such as the PSO algorithm, operators such as intersection and mutation are not used. These algorithms require coding, encoding, and decoding. As a result, they have better speed and accuracy results than the basic evolution algorithms. Generally, these algorithms divide a problem-solving space into several problem-solving subspaces. In each subspace, they seek to optimize the parameters related to the objective function and finally find the general solution by putting together the optimal local solutions. According to the local answers in the PSO algorithm, Local Best (x*) and the general optimal answers Global Best (g*) optimize its parameters according to the relationships in the main PSO formula.

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Table 11
Predicted university enrollment values obtained after iterations 1, 99, and 100 using the proposed NTS-GA model.

Iteration	Year	Real enrollment	Entropy	Optimal entropy	Predicted enrollment	AFER%
1	1971	13,055	0.85	0.87	13362.18	0.167204
	1972	13,563	0.89	0.895	13639.2	
	1973	13,867	0.9	0.92	14175.16	
	1974	14,696	0.94	0.95	14852.34	
	1975	15,460	0.96	0.966667	15567.36	
	1976	15,311	0.96	0.966667	15417.33	
	1977	15,603	0.97	0.9725	15643.21	
	1978	15,861	0.97	0.9725	15901.88	
	1979	16,807	0.99	0.99	16,807	
	1980	16,919	0.99	0.99	16,919	
	1981	16,388	0.98	0.976667	16332.26	
	1982	15,433	0.96	0.966667	15540.17	
	1983	15,497	0.97	0.9725	15536.94	
	1984	15,145	0.96	0.966667	15250.17	
	1985	15,163	0.96	0.966667	15268.3	
	1986	15,984	0.98	0.976667	15929.63	
	1987	16,859	0.99	0.99	16,859	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	
99	1971	13,055	0.84	0.86	13365.83	0.167001
	1972	13,563	0.88	0.89	13717.13	
	1973	13,867	0.9	0.92	14175.16	
	1974	14,696	0.94	0.95	14852.34	
	1975	15,460	0.96	0.96	15,460	
	1976	15,311	0.96	0.96	15,311	
	1977	15,603	0.97	0.98	15763.86	
	1978	15,861	0.97	0.98	16024.52	
	1979	16,807	0.99	0.99	16,807	
	1980	16,919	0.99	0.99	16,919	
	1981	16,388	0.98	0.97	16220.78	
	1982	15,433	0.96	0.96	15,433	
	1983	15,497	0.96	0.96	15,497	
	1984	15,145	0.95	0.956667	15251.28	
	1985	15,163	0.95	0.956667	15269.41	
	1986	15,984	0.97	0.98	16148.78	
	1987	16,859	0.99	0.99	16,859	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	
100	1971	13,055	0.84	0.86	13365.83	0.167001
	1972	13,563	0.88	0.89	13717.13	
	1973	13,867	0.9	0.92	14175.16	
	1974	14,696	0.94	0.95	14852.34	
	1975	15,460	0.96	0.96	15,460	
	1976	15,311	0.96	0.96	15,311	
	1977	15,603	0.97	0.98	15763.86	
	1978	15,861	0.97	0.98	16024.52	
	1979	16,807	0.99	0.99	16,807	
	1980	16,919	0.99	0.99	16,919	
	1981	16,388	0.98	0.97	16220.78	
	1982	15,433	0.96	0.96	15,433	
	1983	15,497	0.96	0.96	15,497	
	1984	15,145	0.95	0.956667	15251.28	
	1985	15,163	0.95	0.956667	15269.41	
	1986	15,984	0.97	0.98	16148.78	
	1987	16,859	0.99	0.99	16,859	
	1988	18,150	1	1	18,150	
	1989	18,970	1	1	18,970	
	1990	19,328	1	1	19,328	
	1991	19,337	1	1	19,337	
	1992	18,876	1	1	18,876	

Table 12
Predicted university enrollment values obtained after iterations 1, 99, and 100 using the proposed NTS-PSO model.

Iteration	Year	Real enrollment	Entropy	Optimal entropy	Predicted enrollment	AFER%
1	1971	13,055	0.87	0.885	13280.08621	0.171761849
	1972	13,563	0.9	0.91	13713.7	
	1973	13,867	0.92	0.94	14168.45652	
	1974	14,696	0.96	0.97	14849.08333	
	1975	15,460	0.98	0.98	15,460	
	1976	15,311	0.97	0.9775	15429.38402	
	1977	15,603	0.98	0.98	15,603	
	1978	15,861	0.98	0.98	15,861	
	1979	16,807	1	0.996666667	16750.97667	
	1980	16,919	1	0.996666667	16862.60333	
	1981	16,388	0.99	0.992	16421.10707	
	1982	15,433	0.98	0.98	15,433	
	1983	15,497	0.98	0.98	15,497	
	1984	15,145	0.97	0.9775	15262.10052	
	1985	15,163	0.97	0.9775	15280.23969	
	1986	15,984	0.99	0.992	16016.29091	
	1987	16,859	1	0.996666667	16802.80333	
	1988	18,150	1	0.996666667	18089.5	
	1989	18,970	1	0.996666667	18906.76667	
	1990	19,328	0.99	0.992	19367.04646	
	1991	19,337	0.99	0.992	19376.06465	
	1992	18,876	1	0.996666667	18813.08	
99	1971	13,055	0.72	0.74	13417.63889	0.164124731
	1972	13,563	0.76	0.77	13741.46053	
	1973	13,867	0.78	0.805	14311.45513	
	1974	14,696	0.83	0.845	14961.59036	
	1975	15,460	0.86	0.86	15,460	
	1976	15,311	0.86	0.86	15,311	
	1977	15,603	0.87	0.875	15692.67241	
	1978	15,861	0.88	0.903333333	16281.55682	
	1979	16,807	0.91	0.915	16899.34615	
	1980	16,919	0.92	0.923333333	16980.30072	
	1981	16,388	0.9	0.88	16023.82222	
	1982	15,433	0.86	0.86	15,433	
	1983	15,497	0.86	0.86	15,497	
	1984	15,145	0.85	0.86	15323.17647	
	1985	15,163	0.85	0.86	15341.38824	
	1986	15,984	0.88	0.903333333	16407.81818	
	1987	16,859	0.92	0.923333333	16920.08333	
	1988	18,150	0.95	0.955	18245.52632	
	1989	18,970	0.96	0.965	19068.80208	
	1990	19,328	0.97	0.966666667	19261.58076	
	1991	19,337	0.97	0.966666667	19270.54983	
	1992	18,876	0.96	0.965	18974.3125	
100	1971	13,055	0.72	0.74	13417.63889	0.164124731
	1972	13,563	0.76	0.77	13741.46053	
	1973	13,867	0.78	0.805	14311.45513	
	1974	14,696	0.83	0.845	14961.59036	
	1975	15,460	0.86	0.86	15,460	
	1976	15,311	0.86	0.86	15,311	
	1977	15,603	0.87	0.875	15692.67241	
	1978	15,861	0.88	0.903333333	16281.55682	
	1979	16,807	0.91	0.915	16899.34615	
	1980	16,919	0.92	0.923333333	16980.30072	
	1981	16,388	0.9	0.88	16023.82222	
	1982	15,433	0.86	0.86	15,433	
	1983	15,497	0.86	0.86	15,497	
	1984	15,145	0.85	0.86	15323.17647	
	1985	15,163	0.85	0.86	15341.38824	
	1986	15,984	0.88	0.903333333	16407.81818	
	1987	16,859	0.92	0.923333333	16920.08333	
	1988	18,150	0.95	0.955	18245.52632	
	1989	18,970	0.96	0.965	19068.80208	
	1990	19,328	0.97	0.966666667	19261.58076	
	1991	19,337	0.97	0.966666667	19270.54983	
	1992	18,876	0.96	0.965	18974.3125	

$$\mathbf{v_i(t+1) = v_i(t) + c_1 r_1(t)\ (p_{best} - x_i(t)) + c_2 r_2(t)\ (g_{best} - X_i(t))} \tag{20}$$

$$\mathbf{X_i(t+1) = X_i(t) + v_i(t+1)} \tag{21}$$

r_1 and r_2 are random numbers in the interval [0, 1] generated in each run.

$$\mathbf{X(0) = x_{min} + R_i(x_{max} - x_{min})} \tag{22}$$

R_i is a random number between [0, 1], and V_{max} is equal to a random number in the minimum and maximum range of the initial parameters of the problem.

If the value of v_i exceeds V_{max} at any moment, we set its value equal to V_{max}, and if the value of v_i becomes less than V_{min} at any moment, we set its value equal to V_{min}.

c_1 and c_2 parameters are for the influence of p_{best} or g_{best} results on the problem data. If the value of $c_1 \gg c_2$ means that the algorithm is more inclined towards local results, the value of $c_2 \gg c_1$ means that the algorithm is more inclined towards general results. To balance the local and general results, we set the parameters c_1 and c_2 in the range (Taghipourian et al., 2021; Kuranga et al., 2023) so that c_1 and c_2 have equal values. The best choice is $c_1+c_2 = 4$, where c_1 and c_2 have almost equal values.

If we consider the values of c_1 and c_2 equal to 1, the particle's movement in the local responses will be somewhat slowed. As a result, it will hurt the general results (the particle has no noticeable movement in an interval.) But if the value of c_1 is close to the number 2 and the value of c_2 close to 1, the algorithm has many jumps in the local response space, but there are no noticeable changes in the general response space. Conversely, if we set c_2 equal to 2 and c_1 close to 1, the algorithm has weak changes in the local response space. But there are many changes in the space of general answers. These two modes prevent convergence in the algorithm. For this reason, it is better to give a balance between parameters c_1 and c_2 in the movement of particles.

4. Result analysis and validation

This section presents the prediction results of the university enrollment data, the TAIFEX index, and the TSEC weighted index dataset. We determine their accuracy by comparing the proposed models' forecasts with existing models based on the AFER index (see Table 12).

4.1. Prediction of university enrollment data set

To validate the proposed hybrid models (NTS-GA, NTS-PSO, and NTS-QOA), we compare them with well-known time series forecasting standard hybrid models such as the FTS-GA model (Chen and Chung, 2006), FTS-PSO model_1 (Kuo et al., 2009), FTS-PSO_2 model (Aladag et al., 2012), and FTS SA model (Radmehr and Gharneh, 2012). The predicted values of enrollment data obtained through the proposed models and the considered combined criteria models are shown in Table 13 regarding AFER.

Table 13 shows that the proposed combined models have an AFER of 0.166%, 0.167%, and 0.164%, respectively. In comparison, existing hybrid models like FTS GA model (Chen and Chung, 2006), FTS-PSO model_1 (Kuo et al., 2009), FTS PSO model_2 (Aladag et al., 2012), and FTS SA model (Radmehr and Gharneh, 2012) have AFERs of 2.19%, 3.11%, 1.58%, and 0.68% respectively. The proposed models' prediction accuracy is significantly better than the existing hybrid models. Additionally, a convergence rate analysis was carried out to compare the performance of the proposed models. As depicted in Fig. 1, the proposed model converges towards the optimal result much faster than the existing hybrid models.

Figs. 2–4 compare the actual and predicted enrollment based on the proposed models. Accordingly, it is apparent that the estimated enrollment closely matches the actual enrollment.

4.2. Prediction of TAIFEX index data set

The proposed NTS-GA, NTS-PSO, and NTS-QOA models were used to forecast the TAIFEX index data set from 3/8/1998–9/30/1998. The predicted results are shown in Table 14. In this table, the predicted results of the proposed models are compared with the existing ones. In comparison, it was observed that the proposed models performed better than the previous models.

Table 15 compares the results of the proposed NTS-QOA, NTS-QOA, and NTS-GA models and the prediction of the TAIFEX index dataset for the period between 1/19/2006 and 12/21/2006. The proposed models exhibit a significantly smaller AFER than the existing models.

4.3. Prediction of TSEC weighted index data sets

The proposed models have been verified by forecasting the TSEC weighted index from 2003 to 2004. The predicted results of the proposed models from 2003 to 2004 are compared with the previous works. Table 16 shows the comparison of other models for forecasting the TSEC weighted index from the period of 2003–2004, considering the average AFER. The proposed models have a much smaller AFER than the existing models.

Fig. 5 shows that the predicted TSEC-weighted index values closely align with the actual TSEC-weighted index values.

4.4. Statistical analysis of the results

The efficiency of the proposed NTS-QOA has been evaluated using statistical parameters that include observed and predicted mean deviation and standard deviation, correlation coefficient, and Theil's U statistic based on Eqns. 23–26.

$$\overline{\mathbf{M}} = \frac{\sum_{i=1}^{n} \mathrm{Act_i}}{\mathbf{n}} \tag{23}$$

$$\mathbf{S_d} = \sqrt{\frac{\mathbf{1}}{\mathbf{n}} \sum_{\mathbf{i=1}}^{\mathbf{n}} (\mathrm{Act_i} - \overline{\mathbf{M}})^2} \tag{24}$$

$$\mathrm{CC} = \frac{n \sum \mathrm{Act_i}\, \boldsymbol{Fore_i} - (\sum \mathrm{Act_i})(\sum \mathrm{Fore_i})}{\sqrt{n \sum \mathrm{Act_i^2} - (\sum \mathrm{Act_i})^2}\sqrt{n \sum \mathrm{Fore_i^2} - (\sum \mathrm{Fore_i})^2}} \tag{25}$$

$$\mathbf{U} = \frac{\sqrt{\sum_{i=1}^{n} (\mathrm{Act_i} - \mathrm{Fore_i})^2}}{\sqrt{\sum_{i=1}^{n} \mathrm{Act_i^2}} + \sqrt{\sum_{i=1}^{n} \mathrm{Fore_i^2}}} \tag{26}$$

where $Fore_i$ and Act_i denote the predicted and actual values for a particular day \ year i, respectively, and n is the total number of days \ years to be predicted. In Eqns. (23) and (24), (Act_1, Act_2, …, Act_n) represent the real values of the time series, and $\overline{M}$ is the average value of all these time series values. If the average and standard deviations of the actual time series values are close to the average and standard deviation of the predicted time series values, then the model's performance can be considered suitable for prediction accuracy.

In Eqn. (25), the correlation coefficient (CC) value must be between [−1, +1], that is, $1 \le CC \le +1$. A model can be considered good if its CC value is greater than or equal to 0.8. In Eqn. (26), the value of U should be between [0, 1]. For good prediction accuracy, the value of U should be close to zero.

All the university enrollment prediction statistics, TAIFEX index, and TSEC weighted index datasets based on the proposed NTS-PSO, NTS-GA, and NTS-QOA models are shown in Tables 17–19. It can be seen from these tables that the averages of actual and predicted registrations are very close to each other. The minimum difference between the real and predicted averages indicates good forecast accuracy in all cases.

The difference between the actual and predicted standard deviation in the case of datasets is minimal, indicating that the proposed models'

Table 13
Comparing the proposed models with the existing ones (the university registration data set).

Year	Real enrollment	Model of (Lee and Chou, 2004)	Model of (Cheng et al., 2006)	Model of (Wong et al., 2010)	Model of (Qiu et al., 2011)	Model of (Gangwar and Kumar, 2012)	Model of (Liu, 2007)	Model of (Singh and Borah, 2013) (Singh and Dhiman, 2018)	Model of (Huarng, 2001b)	Model of (Aladag et al., 2012)	Model of (Kuo et al., 2009)	Model of (Chen and Chung, 2006)	Model of (Radmehr and Gharneh, 2012)	NTS-QOA	NTS-GA	NTS-PSO
1971	13,055	–	–	–	–	–	–	–	–	–	–	–	–	13373.41463	13365.83	13417.64
1972	13,563	14,000	14,025	15,430	14,195	13,563	13,500	13,563	13,563	13,480.38	14,000	–	13,706	13720.7093	13717.13	13741.46
1973	13,867	14,000	14,568	15,430	14,424	13,500	13,800	13,867	13,500	13,480.38	14,000	14,146	13,706	14182.15909	14175.16	14311.46
1974	14,696	14,000	14,568	15,430	14,593	14,500	14,700	14,696	14,500	14,780.95	14,000	14,878	14,749	14935.6087	14852.34	14961.59
1975	15,460	15,500	15,654	15,430	15,589	15,500	15,600	15,425	15,500	15,474.17	15,500	14,878	15,341	15427.45263	15460.00	15460.00
1976	15,311	16,000	15,654	15,430	15,645	15,500	15,400	15,420	15,466	15,523.89	16,000	15,609	15,346	15433.16223	15311.00	15311.00
1977	15,603	16,000	15,654	15,430	15,634	15,500	15,750	15,420	15,392	15,609.76	16,000	15,609	15,346	15570.15158	15763.86	15692.67
1978	15,861	16,000	15,654	15,430	16,100	15,500	15,400	15,923	15,549	15,562.98	16,000	16,214	15,923	16081.29167	16024.52	16281.56
1979	16,807	16,000	16,197	16,889	16,188	16,807	16,800	16,862	16,433	16,751.87	16,000	16,214	16,839	16849.875	16807.00	16899.35
1980	16,919	16,813	17,283	16,871	17,077	16,919	17,100	17,192	16,656	16,837.83	16,833	16,818	17,046	16962.16071	16919.00	16980.30
1981	16,388	16,813	17,283	16,871	17,105	16,500	17,100	17,192	16,624	16,837.83	16,833	16,818	16,400	16219.05155	16220.78	16023.82
1982	15,433	16,789	16,197	15,447	16,369	15,500	15,300	15,425	15,556	15,901.87	16,833	15,609	15,455	15400.50947	15433.00	15433.00
1983	15,497	16,000	15,654	15,430	15,643	15,500	15,750	15,420	15,524	15,524.96	16,000	15,609	15,346	15464.37474	15497.00	15497.00
1984	15,145	16,000	15,654	15,430	15,648	15,500	15,400	15,420	15,497	15,527.33	16,000	14,146	15,346	15265.83777	15251.28	15323.18
1985	15,163	16,000	15,654	15,430	15,622	15,500	15,300	15,627	15,305	15,492.78	16,000	14,146	15,341	15283.98138	15269.41	15341.39
1986	15,984	16,000	15,654	15,430	15,623	15,984	15,750	15,627	15,308	15,570.69	16,000	16,818	15,923	16,206	16148.78	16407.82
1987	16,859	16,000	16,197	16,889	16,231	16,859	16,800	16,862	16,402	16,906.56	16,000	16,818	16,839	16902.00765	16859.00	16920.08
1988	18,150	16,813	17,283	16,871	17,090	18,500	17,100	17,192	18,500	16,837.83	16,833	17,992	18,007	18,150	18150.00	18245.53
1989	18,970	19,000	18,369	19,333	18,325	18,500	18,900	18,923	18,534	19,144.40	19,000	19,126	19,059	18,970	18970.00	19068.80
1990	19,328	19,000	19,454	19,333	19,000	19,337	19,200	19,333	19,345	19,144.40	19,000	19,126	19,197	19,328	19328.00	19261.58
1991	19,337	19,000	19,454	19,333	19,000	19,500	19,050	19,136	19,423	19,144.40	19,000	19,126	19,197	19,337	19337.00	19270.55
1992	18,876	–	–	19,333	19,000	18,704	19,050	19,136	18,752	19,144.40	19,000	19,126	19,059	18,876	18876.00	18974.31
AFER	–	7.83	7.30	2.75	2.65	1.44	1.32	1.19	0.96	1.58	3.11	2.19	0.68	0.166	0.167	0.164

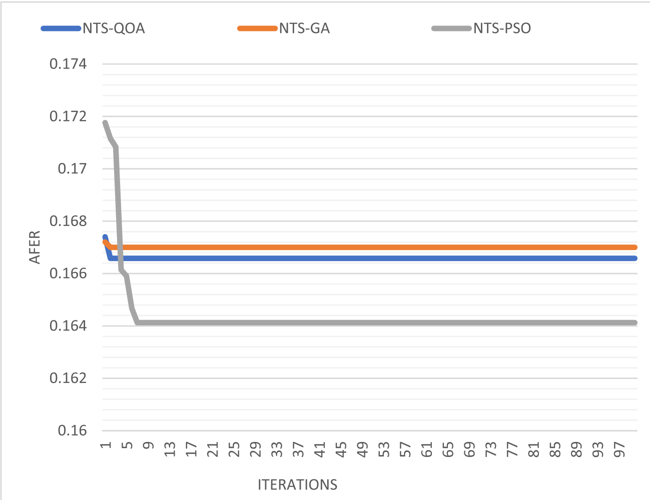

Fig. 1. AFER curve and convergence of the proposed models.

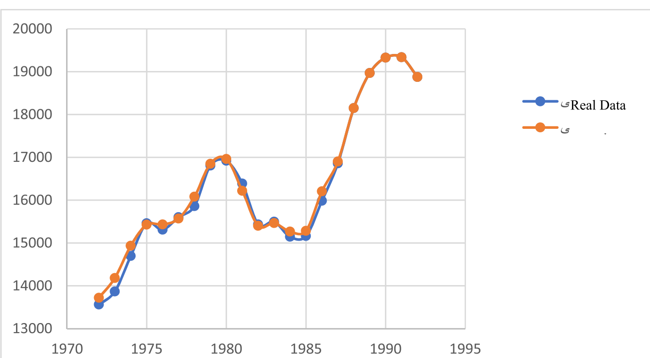

Fig. 2. Comparison of actual with predicted enrollment based on the proposed NTS-QOA model.

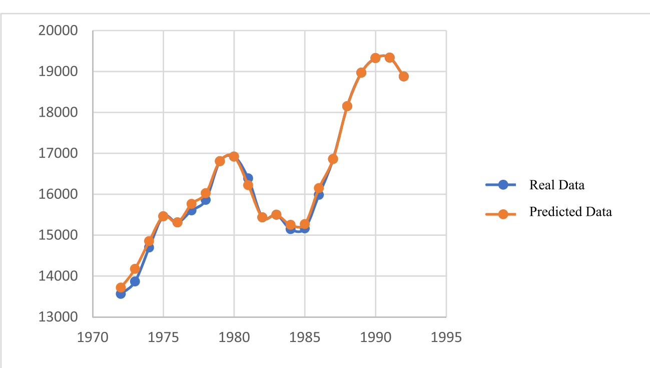

Fig. 3. Comparison of actual with predicted enrollment based on the proposed NTS-GA model.

prediction accuracy is good. The correlation coefficients between the real and predicted values in this data set also show the efficiency of the proposed models.

Theil's U statistic for all these data sets is close to zero, which shows that the proposed models are effective. Each statistic, therefore, supports the proposed models, and the TAIFEX index dataset and the TSEC weighted index dataset look very convincing, with excellent performance across all university enrollment cases.

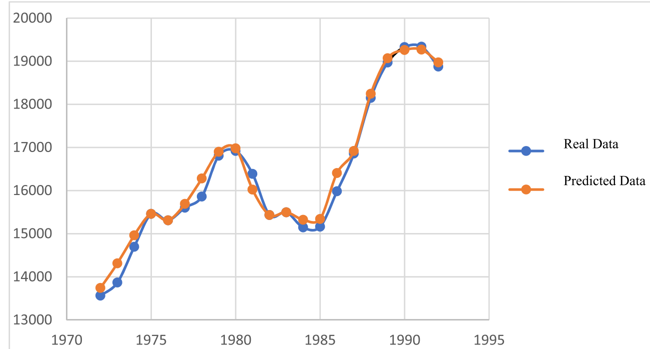

Fig. 4. Comparison of actual with predicted enrollment based on the proposed NTS-PSO model.

5. Conclusion

In this research, we sought a new hybrid model for predicting time series in the neutrosophic environment. Neutrosophic set theory was used to represent the time series data set. This form of representation is called NTS and is mainly used to model time series data sets. In the present study, with the help of the concept of entropy, NERs were created between NTS representations of time series. These NERs were used in the de-fuzzification operation to obtain the prediction results.

This study showed that the prediction accuracy of the NTS modeling method relies more on the optimal selection of the reference set and its corresponding distances. This research problem was solved using the integration of three optimization algorithms, GA, QOA, and PSO, with the NTS modeling approach. The combination of QOA, GA, and PSO in this NTS modeling method helped to search for the overall optimal solution for the reference set and its corresponding distances from the list of locally optimal solutions. Then, the proposed NTS QOA, NTS-GA, and NTS-PSO models were verified by effectively predicting three different data sets: the University of Alabama enrollment, the TAIFEX index, and the TSEC index.

Prediction results showed that the proposed models perform better than other existing models. Among the proposed models, the NTS-PSO model performs better than the proposed NTS-QOA and NTS-GA models regarding convergence speed and minimum AFER parameters.

The study's approach, combining neutrosophic logic and optimization algorithms, can significantly improve forecasting accuracy, practically impacting various industries, including finance, energy, and education, where accurate predictions are crucial for decision-making and planning. Using neutrosophic logic provides a more comprehensive way of understanding and handling uncertainty in time series data. This insight can lead to better decision support systems as decision-makers gain a deeper understanding of the complexities involved in the data.

The study's demonstration of the approach's versatility across different datasets suggests that it can be applied in various domains. This insight can inspire researchers and practitioners to explore the application of neutrosophic logic and optimization algorithms in fields beyond those presented in the study. The study's acknowledgment of limitations and recommendations for future research, such as deep learning and parallel processing, provide a roadmap for further exploration. This insight guides researchers toward areas where improvements can be made, particularly in handling large datasets. The inclusion of Smarandache's philosophical foundation for neutrosophic logic demonstrates the relevance of philosophical perspectives in addressing real-world problems. This insight may encourage researchers to explore interdisciplinary approaches to solving complex issues.

Several potential limitations of the study can be mentioned as follows.

Table 14
Comparison between the proposed and existing models (for the TAIFEX index data set).

Date	Real data	Model of (Lee et al., 2006)	Model of (Lee et al., 2006)	Model of (Lee et al., 2008)	Model of (Huarng et al., 2007)	Model of (Kuo et al., 2009)	Model of (Kuo et al., 2010)	NTS-QOA	NTS-GA	NTS-PSO
March 08, 1998	7552	–	–	–	–	–	–	7513.07	7513.47	7512.67
April 08, 1998	7560	7450	7450	–		–	–	7521.03	7521.43	7520.63
May 08, 1998	7487	7450	7450	–	–	–	–	7481.43	7487.00	7479.84
June 08, 1998	7462	7500	7450	7450	–	–	7452.54	7456.45	7462.00	7454.86
July 08, 1998	7515	7500	7500	7550	–	–	7518.11	7476.26	7476.66	7475.86
October 08, 1998	7365	7450	7450	7350	–	–	7359.49	7359.52	7365.00	7357.95
November 08, 1998	7360	7300	7350	7350	–	–	7359.49	7354.52	7360.00	7352.96
December 08, 1998	7330	7300	7300	7350	7329	7320.77	7331.62	7324.55	7330.00	7322.99
8/13/1998	7291	7300	7350	7250	7289.5	7289.56	7285.63	7285.58	7291.00	7284.02
8/14/1998	7320	7183.33	7100	7350	7329	7320.77	7331.62	7314.55	7320.00	7313.00
8/15/1998	7300	7300	7350	7350	7289.5	7289.56	7291.67	7294.57	7300.00	7293.01
8/17/1998	7219	7300	7300	7250	7215	7222.19	7217.15	7213.63	7219.00	7203.64
8/18/1998	7220	7183.33	7100	7250	7215	7222.19	7217.15	7214.63	7220.00	7204.64
8/19/1998	7285	7183.33	7300	7250	7289.5	7289.56	7285.63	7279.58	7285.00	7278.03
8/20/1998	7274	7183.33	7100	7250	7289.5	7289.56	7279.59	7268.59	7274.00	7267.04
8/21/1998	7225	7183.33	7100	7250	7215	7222.19	7217.15	7219.62	7225.00	7209.63
8/24/1998	6955	7183.33	7100	6950	6949.5	6952.76	6950.85	6951.11	6906.70	6938.97
8/25/1998	6949	6850	6850	6950	6949.5	6952.76	6941.88	6945.11	6900.74	6932.99
8/26/1998	6790	6850	6850	6750	6796	6800.07	6784.34	6786.20	6796.81	6802.30
8/27/1998	6835	6775	6650	6850	6848	6850.12	6843.35	6831.17	6841.85	6819.25
8/28/1998	6695	6850	6750	6650	6698.5	6713.46	6700.46	6705.28	6701.71	6707.13
8/29/1998	6728	6750	6750	6750	6726	6713.46	6721.85	6738.33	6734.74	6740.19
8/31/1998	6566	6775	6650	6550	6569.5	6568.29	6562.63	6551.73	6531.07	6529.92
September 1, 1998	6409	6450	6450	6450	6417	6416.74	6402.61	6395.07	6426.23	6426.80
September 2, 1998	6430	6450	6550	6450	6417	6416.74	6417.14	6416.02	6447.28	6447.86
September 3, 1998	6200	6450	6350	6250	6205	6195	6191.69	6268.89	6233.70	6234.83
September 4, 1998	6403.2	6450	6450	6450	6417	6416.74	6402.61	6389.28	6420.41	6420.99
September 5, 1998	6697.5	6150	6550	6650	6698.5	6713.46	6700.46	6707.79	6704.21	6709.63
September 7, 1998	6722.3	6750	6750	6750	6726	6713.46	6721.85	6732.63	6729.04	6734.48
September 8, 1998	6859.4	6775	6850	6850	6848	6850.12	6852.31	6855.56	6866.28	6843.59
September 9, 1998	6769.6	6850	6750	6750	6763	6767.08	6770.64	6765.81	6776.39	6781.86
September 10, 1998	6709.75	6775	6650	6750	6726	6713.46	6711.15	6720.06	6716.48	6721.91
September 11, 1998	6726.5	6775	6850	6750	6726	6713.46	6721.85	6736.83	6733.24	6738.69
9/14/1998	6774.55	6775	6850	6817	6763	6767.08	6784.34	6770.76	6781.34	6786.82
9/15/1998	6762	6775	6650	6817	6763	6767.08	6770.64	6758.21	6768.78	6774.25
9/16/1998	6952.75	6775	6850	6817	6949.5	6952.76	6950.85	6948.86	6904.47	6936.73
9/17/1998	6906	6850	6950	6950	6904.5	6905.44	6903.03	6902.13	6912.92	6890.09
9/18/1998	6842	6850	6850	6850	6848	6850.12	6843.35	6838.17	6848.86	6826.24
9/19/1998	7039	6850	6950	7050	7064	7033.83	7042.6	7001.95	6990.12	7024.02
9/21/1998	6861	6850	6850	6850	6848	6850.12	6852.31	6857.16	6867.88	6845.19
9/22/1998	6926	6850	6950	6950	6904.5	6905.44	6932.9	6922.12	6877.90	6910.04
9/23/1998	6852	6850	6850	6850	6848	6850.12	6852.31	6848.16	6858.87	6836.21
9/24/1998	6890	6850	6950	6850	6904.5	6905.44	6891.1	6886.14	6896.91	6874.12
9/25/1998	6871	6850	6850	6850	6848	6850.12	6861.26	6867.15	6877.89	6855.17
9/28/1998	6840	6850	6750	6850	6848	6850.12	6843.35	6836.17	6846.86	6824.24
9/29/1998	6806	6850	6750	6850	6796	6800.07	6798.04	6802.19	6812.82	6818.33
9/30/1998	6787	6850	6750	6750	6796	6800.07	6784.34	6783.20	6793.80	6799.30
AFER%	–	0.88	1/02	0.43	0.11	0.12	0.097	0.081	0.081	0.081

Table 15
Comparison between the proposed model and existing ones (for the TAIFEX index data set).

Date	Real data	Model of (Lee et al., 2006)	Model of (Lee et al., 2006)	Model of (Lee et al., 2008)	Model of (Huarng et al., 2007)	Model of (Kuo et al., 2009)	Model of (Kuo et al., 2010)	Proposed NTS-QOA	Proposed NTS-GA	Proposed NTS-PSO
1/19/2006	7649	7650	7650	7650	7650	7650	7650	7569.32	7609.57	7565.86
2/16/2006	7270	7279	7279	7279.5	7279.5	7271.5	7273.5	7231.33	7232.14	7229.61
3/16/2006	7002	7009	7009	7005	7005	7005	7005	6926.71	6957.78	6923.33
4/20/2006	6926	6910	6910	6916	6916	6916	6916	6850.72	6882.26	6886.65
5/18/2006	6750	6739	6739	6749	6741	6741	6741	6676.63	6678.19	6646.55
6/22/2006	6418	6428.5	6428.5	6428.5	6428.5	6418.5	6418.5	6514.15	6487.76	6518.67
7/20/2006	6413	6433.5	6423.5	6423.5	6421.5	6421.5	6421.5	6509.07	6482.71	6513.60
8/17/2006	6998	6988.1	6988.1	6988.1	6988.1	6988.1	6998.1	6922.75	6953.80	6919.37
9/21/2006	7064	7088	7088	7088	7088	7088	7088	6988.04	7019.39	6984.63
10/19/2006	6519	6538.5	6538.5	6538.5	6538.5	6538.5	6538.5	6555.22	6495.63	6556.90
11/16/2006	6656	6678	6678	6678	6668	6661	6661	6619.43	6632.14	6553.99
12/21/2006	6491	6471	6491	6491	6491	6491	6491	6527.06	6561.55	6592.82
AFER%	–	0.23	0.19	0.16	0.16	0.12	0.11	0.11	0.11	0.10

Table 16
Comparison between the proposed model and existing ones (for the TSEC weighted index data set).

Model	2003	2004	AFER
Huarng et al. (Singh et al., 2018)	5.41	6.87	6.14
Bivariate model (Qiao et al., 2022)	8.67	8.97	8.82
Univariate model (Qiao et al., 2022)	11.5	7.68	9.59
AR-1 (Duan et al., 2022)	6.09	5.69	5.89
AR-2 (Duan et al., 2022)	5.19	4.94	5.07
Jiang et al. (Jiang et al., 2017)	4.68	4.85	4.77
Gupta et al. (Gupta et al., 2018)	2.18	2.15	2.17
Singh et al. (Singh and Dhiman, 2018)	1.17	2.15	1.66
Proposed Model-1 (NTS-QOA)	0.126	0.055	0.09
Proposed Model-1 (NTS-GA)	0.126	0.054	0.09
Proposed Model-1 (NTS-PSO)	0.126	0.055	0.09

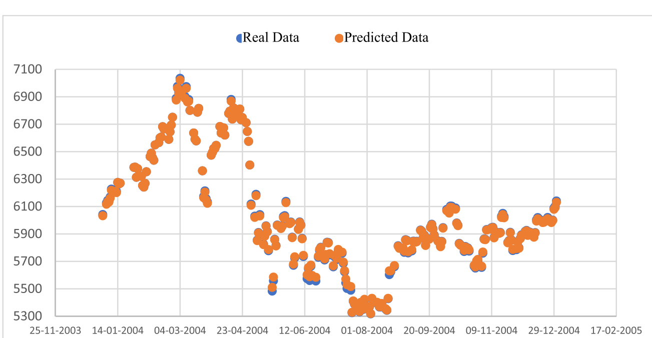

Fig. 5. The actual and predicted weighted index of TSEC for 2004 based on the proposed NTS-PSO model.

- Computational Intensity: The study acknowledges that when dealing with large datasets, the execution speed of the algorithm decreases significantly. The optimization algorithms' computational intensity, especially when combined, can be a limiting factor, particularly when dealing with extensive time series data. This limitation can make the approach less practical for very large datasets without efficient computational resources.
- Limited Generalization: While the study demonstrates the effectiveness of the proposed NTS models on the specific datasets used (university enrollment, financial indices, stock exchange data), its generalizability to other domains and datasets may be limited. It remains to be seen how well the model performs in diverse scenarios beyond the examples provided. The model may not perform equally well in all situations, and the choice of datasets can influence the perceived success of the approach.
- Hybrid Model Selection: The study introduces a hybrid model (NTS-QOA, -GA, and -PSO) but does not extensively explore the criteria for selecting the optimal combination of optimization algorithms. There may be an element of subjectivity in the selection process, which can affect the model's performance.
- Overfitting Concerns: The study does not explicitly address the potential for overfitting when employing multiple optimization algorithms. Overfitting could lead to models that perform exceptionally well on training data but poorly on new, unseen data.

Based on the identified limitations of the study, several potential future research directions can be proposed.

- Given the computational intensity of the optimization algorithms, future research could focus on developing efficient parallel processing techniques to make the proposed approach more scalable for handling large datasets. It could involve exploring distributed, cloud, or other parallel computing architectures.
- Future research could explore integrating deep learning techniques with Neutrosophic Logic and optimization algorithms to improve forecasting accuracy. It could involve developing neural network architectures specifically designed for time series forecasting in uncertain and complex environments. They may explore the potential of combining the proposed NTS-QOA model with other optimization techniques, such as simulated annealing, ant colony optimization, or reinforcement learning, to enhance forecasting accuracy.
- Future research could conduct extensive real-world case studies and applications across various industries to validate the proposed

Table 17
Statistical analysis of predicted results based on the proposed NTS-QOA model.

Data Set/Parameter	University enrollment (See Table 13)		TAIFEX (See Table 14)		TAIFEX (See Table 15)		TSEC (4-4) (See Table 16)	
	Real	Predicted	Real	Predicted	Real	Predicted	Real	Predicted
$\overline{\mathbf{M}}$	16194.18	16.194	6964.25	6942	6846.3	6846.3	5597.84	5586.7
$\mathbf{S_d}$	1816.49	1747/7	331.58	328.03	378.38	362.27	515.16	578
CC	0.9983		0.999		0.0997		0.999	
Theil's U	0.0063		0.0017		0.0072		0.0006	

Table 18
Statistical analysis of predicted results based on the proposed NTS-GA model.

Data Set/Parameter	University enrollment (See Table 13)		TAIFEX (See Table 14)		TAIFEX (See Table 15)		TSEC (4-4) (See Table 16)	
	Real	Predicted	Real	Predicted	Real	Predicted	Real	Predicted
$\overline{\mathbf{M}}$	16194.18	16,194	6964.25	6964.2	6846.33	6846.3	5597.84	5597.85
$\mathbf{S_d}$	1816.49	1774.7	331.58	328.03	378.38	362.27	515.16	514.2
CC	0.9987		0.9983		0.9935		0.999	
Theil's U	0.0056		0.0021		0.0053		0.0022	

Table 19
Statistical analysis of predicted results based on the proposed NTS-PSO model.

Data Set/Parameter	University enrollment (See Table 13)		TAIFEX (See Table 14)		TAIFEX (See Table 15)		TSEC (4-4) (See Table 16)	
	Real	Predicted	Real	Predicted	Real	Predicted	Real	Predicted
$\overline{\mathbf{M}}$	16194.18	16,194	6964.25	6964.2	6846.33	6846.3	5597.84	5597.85
$\mathbf{S_d}$	1816.49	1774.7	331.58	328.03	378.38	362.27	515.16	514.13
CC	0.9953		0.9992		0.9824		0.9999	
Theil's U	0.0094		0.0018		0.0085		0.0009	

approach's practical impact. It would provide valuable insights into the model's effectiveness in different domains. Future studies could perform benchmarking and comparative analyses by testing the proposed approach against a wider range of existing time series forecasting models, including both traditional statistical methods and other advanced techniques. It would help establish the approach's relative strengths and weaknesses.

- Future works may develop interactive decision support systems that utilize the proposed forecasting model to provide real-time decision support for users. It could involve creating user-friendly interfaces that enable domain experts to make informed decisions based on the model's predictions.
- Future studies may investigate the potential for transfer learning in time series forecasting. Can knowledge gained from one domain be effectively transferred to other disciplines, and how can transfer learning techniques be adapted for NTS modeling?

CRediT authorship contribution statement

Seyyed Ahmad Edalatpanah: Data curation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Final draft preparation. **Farnaz Sheikh Hassani:** Investigation, Methodology, Supervision, Writing – original draft, Writing – review & editing, Final draft preparation. **Florentin Smarandache:** Investigation, Methodology, Supervision, Writing – original draft, Writing – review & editing, Final draft preparation. **Ali Sorourkhah:** Investigation, Methodology, Supervision, Writing – original draft, Writing – review & editing, Final draft preparation. **Dragan Pamucar:** Supervision, Writing – original draft, Writing – review & editing, Final draft preparation. **Bing Cui:** Conceptualization, Data curation, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Final draft preparation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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